Transient response in torsional systems with backlash

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Summary

Backlash plays an important role in many engineering applications. A common source of backlash occurs in geared systems due to the clearance between teeth. Backlash in dynamic systems often results in colliding bodies. Colliding bodies can reduce the system’s performance and increase wear. Backlash can be modeled as a piecewise linear phenomenon. The piecewise behavior of the stiffness between two bodies makes the system discontinuous. Although piecewise linear systems have been studied extensively, these studies focused primarily on periodic orbits of oscillators with backlash. Less research has been conducted on the transient dynamic behavior of systems with backlash. The transient dynamics of a system are especially important as they define the behavior of the system due to changes in operating conditions. An example of a problematic effect of transient dynamics in systems with backlash can be found in the drive line of a vehicle. Sudden torque changes such as engine tip-in/tip-out or a change in road conditions can cause the gears to impact. This impact, often referred to as *clunk*, can be heard as a dull sound and can damage the gears. Knowledge of this phenomenon can help in the design of transient engine control strategies.

To study the clunk phenomenon a 4 DOF system with backlash is considered which resembles a low complexity drive line system. A numerical model is used to study the transient dynamics as a result of a reduced drive torque, an engine tip-out. Two different impact modeling techniques are compared. Focus lies on the impacts that occur between the backlash bodies. These impacts have been qualified into combinations of Double Sided and Single Sided Impacts as a function of the static/dynamic loading ratio and the backlash size. Furthermore, the severity of the combined impacts is studied.

The global trend in both numerical and experimental results show that more Double Sided Impacts occur at higher load ratios and smaller backlash gaps. Furthermore, transitions have been observed between Double Sided and Single Sided Impact regions. The results agree with earlier findings and give a more complete overview of the transition between different regions. The severity of the impacts is quantified with the peak-to-peak acceleration of the colliding bodies. Increasing load ratios increase the severity of the impacts. However, there appears to be an extremum in the severity of impacts with increasing backlash size. The observed behavior can explain contradicting statements in previous studies.
Nomenclature

$\theta$: Gear rotation
$J$: Inertia
$k$: Spring constant
$d$: Damping constant
$r$: Radius of backlash gears
$\phi_r$: Backlash size
$\phi_\theta$: Rotational backlash size
$T(t)$: Loading Torque
$T_s$: Static Torque
$T_d$: Dynamic Torque
$\sigma_r$: Load ratio
$\dot{\theta}_{rel}^0$: Velocity before impact
$\dot{\theta}_{rel}^1$: Velocity after impact
$e$: Newton restitution coefficient
$\tau_c$: Characteristic time
$n_c$: Number of periods within characteristic time
$Q_i$: Severity of single impact
$Q_{total}$: Combined severity of impact
Chapter 1

Introduction

Backlash plays an important role in many engineering applications. A common source of backlash occurs in geared systems due to the clearance between teeth. Backlash in dynamic systems often results in colliding bodies. Colliding bodies can reduce the system’s performance and increase wear. Backlash can be modeled as a piecewise linear phenomenon. The piecewise behavior of the stiffness between two bodies makes the system discontinuous. Although piecewise linear systems have been studied extensively, these studies focused primarily on periodic orbits of oscillators with backlash. Less research has been conducted on the transient dynamic behavior of systems with backlash. The transient dynamics of a system are especially important as they define the behavior of the system due to changes in operating conditions. An example of a problematic effect of transient dynamics in systems with backlash can be found in the drive line of a vehicle. Sudden torque changes can cause the gears to impact. An example of such a torque change is an engine tip-out, a step in the applied motor torque. The resulting impact, often referred to as clunk, can be heard as a dull sound and can damage the gears. Knowledge of this phenomenon can help in the design of transient engine control strategies.

1.1 Literature review

Extensive literature exists on vibrations in geared systems. Vibrations can for example occur due to time variable mesh stiffness or backlash. An extensive overview of this field is given by Wang [8]. Most research focuses on the periodic response, stability and bifurcations of such systems. This approach is used to study the periodic response of single and multi degree of freedom geared systems with backlash [11], [10], [4].

One of the main concerns in numerical simulation of systems with backlash is the modeling of impact dynamics. A literature overview of several impact models is given by Gilardi [6]. This overview makes the distinction between discrete and continuous models. An example of a discrete model, namely the restitution coefficient model, is used to study chatter and sticking in MDOF harmonic oscillators [12], [3]. An example of a continuous model is the Hertzian impact model [5].

A problem which is not covered in Wang [8] is the transient response of a geared system as a result of a change in operating conditions. An example of such a change in operating conditions is a sudden drive torque step. Such a step is often named an engine tip-in/tip-out and can cause the gear teeth to impact. This transient phenomenon which is sometimes referred to as clunk is studied by Krenz [9] and by Crowther [2]. In order to quantify the severity of the gear impacts
multiple metrics have been proposed [1]. Two of the proposed metrics are used by Crowther and Singh [7] on a simplified experimental drive line setup with backlash. This report is an extension of the work by Krenz [9] and Crowther/Singh [7] and aims to provide more insight into the nature and severity of impacts as a function of the system and load parameters.

1.2 Objectives

The goal of this project is to quantitify the transient dynamics in geared systems with backlash. The main focus is on impact events resulting from a sudden drive torque change, an engine tip-out, which can cause problems in operational use. The number, nature and severity of impacts is studied with respect to the loading conditions and system parameters.

Subject of this analysis is a 4 DOF system which resembles a low complexity drive line system. A numerical model is developed to study the effect of the system parameters on the transient behavior. Two different impact relations are considered to give the most accurate results. Further, measurements on an experimental setup are used to verify the numerical results.

The objectives of this work are threefold:

- First, quantify the number, nature and severity of impacts resulting from a sudden change in operating conditions.
- Second, explain the contradicting statements of Krenz [9] and Crowther/Singh [7].

This report is organized as follows. First, in chapter 2 the dynamics of a 4 DOF drive line system are presented, different impact relations are considered and the equations of motion are given. Furthermore, the numerical simulation method is discussed. In chapter 3 the results of this numerical model are presented and the sensitivity to different system and loading parameters is studied. Next, in chapter 4 an experimental setup is discussed to verify the numerical results. Chapter 5 presents the conclusions of this research and suggests future directions.
Chapter 2

System Dynamics

This chapter introduces the dynamic system subject of this study. Two different impact modeling techniques with their respective simulation approaches are presented. Next, the equations of motions and model parameters are given.

2.1 Dynamic model

Subject of this study is 4 DOF rotational system. Figure 2.1 shows the dynamic system which is considered. The model has four rotational degrees of freedom resulting in the global coordinate vector $\theta = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4]^T$. These coordinates can globally be related to the elements in a drive line system as depicted in Table 2.1.

There is a backlash gap located between coordinates $\theta_3$ and $\theta_4$. Because the driven inertia is assumed to be orders of magnitude larger than the total drive line inertia the system is grounded at the fourth inertia by a spring with spring constant $k_4$.

The backlash between the third and fourth inertia is a result of the clearance between two gear teeth $\phi_r$. Because both the gears have a radius of $r$ the resulting angular backlash between the two inertias is

$$\phi_g = \frac{\phi_r}{r} \quad (2.1)$$

An input torque is applied to the first inertia which can be related to the motor torque in a drive line system. This input is time dependent and is given by

Table 2.1: Degrees of freedom with their representations in a drive line system

| $\theta_1$ | motor rotation          |
| $\theta_2$ | drive shaft upper flange rotation |
| $\theta_3$ | pinion rotation          |
| $\theta_4$ | ring gear rotation       |
The model is initially loaded with both static and dynamic load. This static load causes inertias 3 and 4 to be in contact with each other. At $t = t_0$ the dynamic load $T_d$ is released to simulate an engine tip-out, a sudden decrease in applied load. The load characteristic is shown graphically in Figure 2.2.

The ratio between the dynamic force and the static force will be referred to as $\sigma_r$ and is given by

$$\sigma_r = \frac{T_d}{T_s} \quad (2.4)$$

The model is assumed to be frictionless in order to study the effect of the piecewise linear behavior of the backlash gap without introducing other non-linearities. However, to account for the frictionless model additional damping is introduced. Due to the high damping coefficients a realistic response is acquired which can be compared with previous experiments [1].
2.2 Model parameters

The nominal model parameters are shown in Table 2.2. These parameters result in the 4 natural frequencies: 3.2 Hz, 16.2 Hz, 30.8 Hz, 258.3 Hz. The lowest natural frequency is approximately 3 Hz and resembles the lowest or shuffle frequency of a drive line system of a car in first gear [7]. This mode corresponds to the global excitation of all the coordinates which is known to occur in transient and can cause gears to impact. All the modes have relatively high damping ($\zeta = 10\% - 20\%$).

Table 2.2: Nominal system parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia</td>
<td>kg m$^2$</td>
<td>0.024, 0.020, 0.020, 0.020</td>
</tr>
<tr>
<td>Stiffness</td>
<td>N/m</td>
<td>31.3, 31.3, 31.3, 31.3</td>
</tr>
<tr>
<td>Damping</td>
<td>Ns/m</td>
<td>0.1, 0.08, 0.04, 0.1</td>
</tr>
<tr>
<td>Angular backlash gap</td>
<td>°</td>
<td>6</td>
</tr>
<tr>
<td>Static load</td>
<td>Nm</td>
<td>$-1$</td>
</tr>
<tr>
<td>Force ratio</td>
<td>[-]</td>
<td>5</td>
</tr>
</tbody>
</table>
2.3 System Dynamics and Numerical Simulation Approach

Impact modeling has been studied for many decades. An overview of these methods is given by Gilardi [6]. Two of these methods are of particular interest, the Restitution Coefficient method and the Indentation Force method. This section introduces the system dynamics of the system shown in Figure 2.1. Furthermore, the numerical simulation approach is presented for both methods.

2.3.1 Restitution Coefficient Model

The Restitution Coefficient Model (RCM) consists out of two states. The state in which inertia $J_3$ and $J_4$ make contact is defined as the sticking state. The state in which inertia $J_3$ and $J_4$ do not make contact is defined as the backlash state. In the backlash state the system dynamics break down into two uncoupled subsystems. The equations of motion can be written as:

\[
\begin{align*}
J_1 \ddot{\theta}_1 &= k_{12} (\theta_2 - \theta_1) + c_{12} (\dot{\theta}_2 - \dot{\theta}_1) - d_1 \dot{\theta}_1 + T_1 \\
J_2 \ddot{\theta}_2 &= -k_{12} (\theta_2 - \theta_1) - c_{12} (\dot{\theta}_2 - \dot{\theta}_1) + k_{23} (\theta_3 - \theta_2) + c_{23} (\dot{\theta}_3 - \dot{\theta}_2) \\
J_3 \ddot{\theta}_3 &= -k_{23} (\theta_3 - \theta_2) - c_{23} (\dot{\theta}_3 - \dot{\theta}_2) + r c_{34} (r \dot{\theta}_4 - r \dot{\theta}_3) \\
J_4 \ddot{\theta}_4 &= -k_4 \dot{\theta}_4 - c_4 \dot{\theta}_4 - r c_{34} (r \dot{\theta}_4 - r \dot{\theta}_3)
\end{align*}
\] (2.5)

These equations can be rewritten into matrix form as

\[
J \ddot{\theta} + C \dot{\theta} + K \theta = T
\] (2.6)

With

\[
J = \begin{bmatrix}
J_1 & 0 & 0 & 0 \\
0 & J_2 & 0 & 0 \\
0 & 0 & J_3 & 0 \\
0 & 0 & 0 & J_4
\end{bmatrix}, \quad
C = \begin{bmatrix}
d_1 + c_{12} & -c_{12} & 0 & 0 \\
-c_{12} & c_{12} + c_{23} & -c_{23} & 0 \\
0 & -c_{23} & c_{23} + r^2 c_{34} & 0 \\
0 & 0 & 0 & c_4 + r^2 c_{34}
\end{bmatrix}, \quad
T = \begin{bmatrix}
T(t) & 0 & 0 & 0
\end{bmatrix}^T.
\]

The zero cross terms between $\theta_3$ and $\theta_4$ in the stiffness and damping matrices indicate that the system decouples.

When inertia $J_3$ and $J_4$ make contact or impact, an instantaneous velocity change is determined using the restitution coefficient method. The condition for such an impact is:

\[
|r(\theta_3 - \theta_4)| \geq 0.5 \phi_r
\] (2.7)

The restitution coefficient method assumes two colliding bodies to be perfectly rigid. The method relates the relative velocity $\dot{\theta}_{rel} = \dot{\theta}_3 - \dot{\theta}_4$ just before impact to the relative velocity just after impact. The time in which this velocity change takes place is infinitesimally small. The velocities just before and after impact are related by:
\[ \dot{\theta}_{rel}^{(0)} = -e\dot{\theta}_{rel}^{(1)} \]  

(2.8)

Where the superscripts 0 and 1 relate to velocities before impact and after impact respectively. The coefficient \( e \) represent the Newton restoration coefficient which is a dimensionless quantity between 0 and 1. A fully elastic impact without energy losses is modeled by a restoration coefficient of 1 while a fully inelastic impact is modeled by a restoration coefficient of 0. The restoration coefficient is not a material property but depends on a combination of material, geometry and velocity. Together with the law of the conservation of momentum this results in a velocity change of both bodies which can be written as:

\[
\begin{align*}
\dot{\theta}_3^{(1)} &= \frac{J_3\dot{\theta}_3^{(0)} + J_4\dot{\theta}_4^{(0)} - eJ_4(\dot{\theta}_3^{(0)} - \dot{\theta}_4^{(0)})}{J_3 + J_4} \\
\dot{\theta}_4^{(1)} &= \frac{J_3\dot{\theta}_3^{(0)} + J_4\dot{\theta}_4^{(0)} + eJ_3(\dot{\theta}_3^{(0)} - \dot{\theta}_4^{(0)})}{J_3 + J_4}
\end{align*}
\]  

(2.9)

Equations (2.5) can now be integrated using the MATLAB ODE45 numerical solver. The event detection option in the ODE environment is used to make sure that the process iterates to the exact time point an impact event occurs. At this time point a velocity change is calculated and the numerical integrations process starts with the new conditions.

A sequence of decaying impacts is often referred to as chatter. This chatter can cause numerical problems as the time between subsequent impacts becomes infinitely small. In this case of complete chatter the two bodies are effectively held together with a zero relative velocity. This phenomenon is called the sticking region [3]. The start of this region is defined as a condition on the relative velocity. If the relative velocity after an impact calculation becomes sufficiently small then the simulation rigidly restrains the motion between the two bodies. The sticking state introduces diagonal terms between inertia \( J_3 \) and \( J_4 \) in both \( K \) and \( D \) matrices which are significantly larger than any other term. This results in the 'coupled' matrices \( K_c \) and \( D_c \). The end of the sticking state is defined as a condition on the force between the two bodies. When the bodies tend to separate (i.e. the force between them changes sign) the simulation uncouples inertia \( J_3 \) and \( J_4 \) and the simulation returns to the backlash state.

A graphical scheme of the numerical simulation process is shown in Figure 2.3. The simulation starts in the sticking state and the initial condition are determined by \( \theta \) with \( \theta = K_c^{-1}T(0) \). \( T_0 \) represents the initial torque vector \( T = \begin{bmatrix} T & 0 & 0 & 0 \end{bmatrix}^T \). The equations of motion are integrated until inertias \( J_3 \) and \( J_4 \) tend to separate. At this point the simulation switches to the dynamics of the backlash state. The uncoupled equations of motion are integrated until an impact is detected. The velocity just after impact is determined using (2.9). After subsequent impacts the relative velocity between the two inertias becomes sufficiently small and the inertias are assumed to be in the sticking region.
2.3.2 Indentation Force Model

The Indentation Force Model (IFM) is based on the insight that colliding bodies can not be considered as perfectly rigid but rather exhibit elastic behavior. This implies that an additional spring and damper is introduced as shown in Figure 2.4 resulting in a finite contact time between the bodies.

The Indentation Force Model consists of two states. The \textit{backlash state} in which the 'uncoupled' Equations (2.6) apply and the \textit{sticking state} in which the spring and damper cause a contact force between the two bodies:

\[
F = F_k(\delta) + F_v(\dot{\delta})
\]  
(2.10)

In which \(\delta = r(\theta_4 - \theta_3 \pm \frac{\theta_5}{2})\) represents the indentation of the spring and \(\dot{\delta} = r(\dot{\theta}_4 - \dot{\theta}_3)\) represents the derivative of the indentation. The relations for \(F_k(\delta)\) and \(F_v(\dot{\delta})\) can for example be linear and given by:

\[
F_k(\delta) = k_{34}\delta \quad \quad \quad F_v(\dot{\delta}) = d_{34}\dot{\delta}
\]  
(2.11)
The equation of motion in the sticking state can now be written as:

\[
\begin{align*}
J_1\ddot{\theta}_1 &= k_{12}(\theta_2 - \theta_1) + c_{12}(\dot{\theta}_2 - \dot{\theta}_1) - d_1\dot{\theta}_1 + T(t) \\
J_2\ddot{\theta}_2 &= -k_{12}(\theta_2 - \theta_1) - c_{12}(\dot{\theta}_2 - \dot{\theta}_1) + k_{23}(\theta_3 - \theta_2) + c_{23}(\dot{\theta}_3 - \dot{\theta}_2) \\
J_3\ddot{\theta}_3 &= -k_{23}(\theta_3 - \theta_2) - c_{23}(\dot{\theta}_3 - \dot{\theta}_2) + r k_{34}(r\dot{\theta}_4 - r\theta_3 \pm \frac{\phi_r}{2}) + r c_{34}(r\dot{\theta}_4 - r\dot{\theta}_3) \\
J_4\ddot{\theta}_4 &= -k_4\dot{\theta}_4 - c_4\dot{\theta}_4 - r k_{34}(r\theta_4 - r\theta_3 \pm \frac{\phi_r}{2}) - r c_{34}(r\dot{\theta}_4 - r\dot{\theta}_3)
\end{align*}
\]

(2.12)

A graphical scheme of the numerical simulation process is shown in Figure 2.5. The simulation starts in the sticking state and the initial condition are determined by solving (2.12) for $\dot{\theta} = 0$, $\ddot{\theta} = 0$ and $T = T_s + T_d$. From these initial conditions the equations of motion are integrated until inertia $J_3$ and $J_4$ lose contact. At this point, the simulation switches to the backlash state, Equation 2.6. The uncoupled equations of motion are integrated up till the point an impact is detected. At this point the process switches back to the sticking state.
Initial conditions

Separation detection

Integrate coupled EOM

Integrate uncoupled EOM

Impact detection

Figure 2.5: Graphical scheme of the numerical process for Indentation Force Model
Chapter 3

Numerical Results

The previous chapter has considered two dynamic models of a 4 DOF system with backlash. Two numerical methods were suggested to model the impacts between the colliding bodies, the Restitution Coefficient Model and the Indentation Force Model. This chapter will compare the simulation results of both models during a transient event. Furthermore, the transient response will be qualified by the resulting number of impacts. Finally the relative severity of these impacts is determined.

3.1 Global response

Figure 3.1 shows the global response of the 4 coordinates $\theta = [\theta_1 \ \theta_2 \ \theta_3 \ \theta_4]^T$ with the nominal system and loading parameters ($\sigma_r = 5$). The system is in equilibrium up till point $t_0 = 0.1s$ at which a step in applied load causes an excitation in the system. The lowest resonance frequency of approximately 3 Hz can clearly been recognized. The vibrational response of the lowest mode is the cause of impacting gears [7].

In order to examine the transient behavior with respect to the backlash gap the relative motion between the teeth on inertia $J_3$ and $J_4$ is examined. This relative coordinate will be referred to as the backlash coordinate and is defined as $x_b = r(\theta_3 - \theta_4)$. Figure 3.2 shows time histories of the backlash coordinate for increasing load steps. A small load ratio ($\sigma_r = 1$) clearly results in a response that remains in the sticking state. At higher load ratios ($\sigma_r = 2$ and $\sigma_r = 5$) more impacts start to occur as the system switches states.

Figure 3.2 shows minor differences between the Restitution Coefficient Model and the Indentation Force Model. In the RCM inertia $J_3$ and $J_4$ are initially rigidly attached. This results in a backlash coordinate displacement of half the backlash gap. Figure 2.4 shows that in the IFM the indentation of the spring results in initial displacement of more than half the backlash gap. Figure 3.3 shows a closeup of the nominal response ($\sigma_r = 5$) at the first impact. This figure shows that in fact multiple impacts occur in both models. This sequence of impacts is often referred to as chatter [3]. After subsequent impacts, inertia $J_3$ and $J_4$ stick together. The resemblance of the two models after tuning of the Newton restitution coefficient $e$ is remarkable. At $e = 0.5$ The RCM essentially predicts the same amount of chatter impacts as the IFM. The main difference between the two models is the timing of these impacts. The IFM chatter impacts appear to have a slight delay due to the flexible behavior of the gear teeth.

The next section will give a qualitative overview of the impacts that occur due to a sudden load step.
In this qualitative study the focus will be on the impacts that occur due to the global response of the system. These impacts are sometimes called primary impacts. Figure 3.3 showed besides a primary impact also impacts due to chatter. These impacts are called secondary impacts. Both models predict the primary impacts equally. Therefore the Indentation Force Model is used to study the effect of changing system and loading parameters.
Figure 3.2: Comparison of the backlash coordinate $x_b$ between the RCM and the IFM. The gray area represents the backlash gap; the state in which $J_3$ and $J_4$ do not make contact with each other. Top: $\sigma_r = 1$ Middle: $\sigma_r = 2$ Bottom: $\sigma_r = 5$
Figure 3.3: Comparison of the backlash coordinate $x_b$ between the RCM and the IFM. The gray zone represents the backlash gap. Closeup of the first impact for an engine tip-out $\sigma_r = 5$

3.2 Double and Single Sided Impacts

The nominal response in Figure 3.2 ($\sigma_r = 5$) shows three primary impacts. These primary impacts can qualitatively be separated into two groups. A Double Sided Impact (DS) consists of a pair of primary impacts on opposite sides of the backlash gap. In this case the entire backlash gap is crossed. Furthermore, a Single Sided Impact (SS) consists of one primary impact. In this case the backlash gap is crossed only partially. The nominal response of Figure 3.2 ($\sigma_r = 5$) consists of 1 Double Sided Impact and 1 Single Sided Impact.

Depending on the applied load step ($\sigma_r$) and the size of the backlash gap there can exist combinations of both single and double sided impacts. Figure 3.4 illustrates the effect of variations in force ratio $\sigma_r$ and backlash gap $\phi_0$ on the number and nature of impacts. This figure is generated by performing numerical simulation on a grid between the force ratio and backlash gap. The different regions represent a combination of Single Sided Impacts and Double Sided Impacts. In this figure NI stands for No Impacts, SS(1) refers to one Single Sided Impact and DS(2)SS(1) refers to two Double Sided Impacts followed by one Single Sided Impact.

The results in this figure agree with earlier findings by Crowther and Singh [7]. However, Figure 3.4 gives a more extensive overview of the parameter space and shows that the boundaries between the regions can be non-linear.

A few aspects of Figure 3.4 can be verified analytically. On the horizontal axis ($\phi_0=0$) of Figure 3.4 the discontinuities disappear as the backlash gap becomes infinitesimally small. The linear system response after the load release at $t_0$ can be described as a function of the first natural frequency, the first damping ratio and the initial conditions. Figure 3.5 shows this graphically. Here, the infinitely small backlash gap is represented by the horizontal axis. The characteristic time $t_c$ is defined as the time point at which the exponential decay crosses the horizontal axis.

The number of periods that fall within the characteristic time gives an indication of the number of impacts that occur. Equation (3.1) gives the number of periods that fall within $t_c$ which is solely a function of the damping ratio $\zeta$ and the load ratio $\sigma_r$. 

\begin{equation} N = \frac{t_c}{t_0} \end{equation}
Figure 3.4: Number and nature of impacts as a function of the load ratio $\sigma_r$ and the backlash gap $\phi_\theta$

Figure 3.5: Linear system response for small backlash size
\[ n_c = \frac{1}{\zeta} \ln(\sigma_r) \]  

(3.1)

No impacts occur as long as \( n_c < 0.5 \). Subsequently, \( 0.5 < n_c > 1.5 \) causes the system to enter the backlash state once. Equation (3.1) can therefore be solved for the load ratio \( \sigma_r \). Table 3.1 summarizes the results. Obviously, this linear approximation is only valid for small backlash sizes. The results of this Table 3.1 agree with the transitions on the horizontal axis of Figure 3.4.

<table>
<thead>
<tr>
<th>Number of impacts</th>
<th>( n_c )</th>
<th>( \sigma_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0-0.5</td>
<td>0-1.46</td>
</tr>
<tr>
<td>1</td>
<td>0.5-1.5</td>
<td>1.46-3.13</td>
</tr>
<tr>
<td>2</td>
<td>1.5-2.5</td>
<td>3.13-6.69</td>
</tr>
<tr>
<td>3</td>
<td>2.5-3.5</td>
<td>6.69-14.32</td>
</tr>
</tbody>
</table>

Table 3.1: Number of impacts and corresponding periods and load ratios

3.3 Impact quantification

In the previous section a qualitative overview of the number and nature of impacts is given for an impulsive event. In this section the qualitative overview will be extended by quantification of the severity of the impacts.

3.3.1 Metric for impact quantification

A number of metrics is developed to quantify impact responses by Crowther and Singh [1]. These include global metrics that consider the complete time history, metrics that are based on the state just prior to impact and metrics that are based on the state just after impact. In this report a metric is used that is based on information just after the impact: the peak-to-peak acceleration. This metric is solely based on primary impacts and therefore neglects the secondary impacts (chatter). However, it is a metric that can simply be measured in both numerical model and experimental setup. The peak-to-peak acceleration of the third inertia \( J_3 \) is used for the quantification. This peak-to-peak acceleration for an impact \( i \) is given by:

\[ Q_i = \text{max}(\ddot{\theta}_3) - \text{min}(\ddot{\theta}_3) \]  

(3.2)

Figure 3.6 shows the time history of \( \ddot{\theta}_3 \) for the nominal system and load parameters (DS(i)SS(i)). In the figure the size of the peak-to-peak accelerations is given graphically.

The \( n \) peak to peak accelerations have to be combined in one single metric. Therefore, the square root of the sum of squared peak-to-peak accelerations is determined:

\[ Q = \sqrt{\sum_{i=1}^{n} Q_i^2} \]  

(3.3)

This metric assures that negative and positive peak to peak values do not cancel.
3.3.2 Impact Severity

In the previous section a metric was defined to quantify the severity of combined impacts. Figure 3.7 shows this metric as a function of the force ratio $\sigma_r$ and the backlash size $\phi_0$. The global trend in this figure seems to be an intuitive one. Increasing force ratio increases overall impact severity. Furthermore, for small backlash sizes the impact severity increases with increasing backlash size. This observation agrees with Krenz’s quote “Clunk severity increases with drive line lash, however not necessarily in a linear relationship”. However Figure 3.7 shows that the impact severity decreases for higher backlash sizes. This agrees with the findings of Crowther and Singh [7]: “For this particular system the dynamics therefore yield a case where reduced lash is leading to a more severe case of clunk(s)

This study into the severity of impact shows a maximum in the combined peak-to-peak accelerations metric. This could be important in reducing the impact responses in drive line systems.

In practical systems a minimal amount of backlash may be essential to assembly and operational use. This causes a lower bound in Figure 3.7. Backlash has other disadvantaged besides impacts. The system is harder to control and therefore less accurate. In practice this will create a lower bound in Figure 3.7. Depending on the expected load ratios one could choose to increase or decrease the backlash size within these bounds.

Figure 3.6: Acceleration response for an engine tip-out $\sigma_r = 5$
Figure 3.7: Peak-to-peak acceleration metric as a function of the load ratio and the backlash size
Chapter 4

Experimental Verification

In the previous chapter the qualitative behavior of a MDOF system with backlash is studied with respect to the number and nature of the impacts. Subsequently, the behavior is quantified by introducing a metric for the severity of the impacts. In this chapter an experimental setup will be discussed which verifies the numerical results qualitatively.

4.1 Experimental setup

The experimental setup is designed to globally resemble the drive line system as shown in Figure 2.1. The experiments aim to verify the numerical results qualitatively and therefore the system parameters do not match. The setup is shown in Figure 4.1 and consists out of four inertial plates which represent the degrees of freedom. The inertia of the plates can be adjusted in order to change the system parameters in further research. Elastic square shafts connect the four discs. The backlash is located between the third and fourth inertia. Figure 4.2 shows a close up of the backlash mechanism which consists of a pin attached to inertia $J_3$ that impacts the slot in inertia $J_4$. The size of the backlash gap has a nominal value of $\phi_0 = 3.6^\circ$ and can be adjusted. The setup is rigidly clamped to simulate the relative large inertia of the vehicle compared to the drive line inertia.

The loading mechanism consist of a spring connected to the first inertia which generates a constant force. The dynamic torque is generated by a mass attached to the first inertia which can be released instantly. An accelerometer is mounted to the third inertia to be able to measure the acceleration at impact.
Figure 4.1: Overview of the experimental setup

Figure 4.2: Closeup of the backlash gap in the experimental setup
4.2 Results

The previous section showed the experimental setup. In this section a number of experiments verify the numerical results given in Chapter 3.

4.2.1 Number and nature of impacts

Figure 4.3 shows the output of the accelerometer mounted to inertia $J_3$. This response is a result of a load step of $\sigma_r = 6.7$ and a backlash gap of $\phi_\theta = 3.6^\circ$. The accelerometer records a voltage as a function of the acceleration. Because only the relative values of the acceleration are considered the accelerometer output in Volts can be used as a measure for the acceleration. The peaks in the response indicate a number of impacts. The peaks are nonsymmetric with respect to the zero acceleration line. The asymmetry is used for qualification of the impacts as initial side impact and opposite side impacts. Initial side impacts occur at the initial side of the backlash and have an acceleration peak biased towards negative acceleration values. Opposite side impacts occur when the backlash gap is crossed and have an acceleration peak biased towards positive acceleration values. Figure 4.3 shows an initial side impact, opposite side impact and a initial side impact subsequently. This sequence of impacts corresponds to one Double Sided Impact and one Single Sided Impact.

![Figure 4.3: Acceleration response of an experiment with load ratio $\sigma_r = 6.7$ and a backlash of $\phi_\theta = 3.6^\circ$.](image)

A number of experiments is conducted with changing ratio $\sigma_r$ and backlash size $\phi_\theta$. Table 4.1 shows the impacts that occurred in these experiments. The results in the table can be compared to the numerical results of Figure 3.4. At small load ratios only Single Sided Impacts occur as can be seen in both the table and the figure. Increasing the backlash size at small load ratios does not change the number and nature of the impacts. Increasing the backlash size at higher load ratios results in a decreasing number of Double Sided Impacts. Transitions from D(1)+S(1) to S(1), from D(2)+S(1) to D(1)+S(2) and from D(3)+S(1) to D(2)+S(1) can be observed. These transitions are also present in the numerical results of Figure 3.4. One transition in Table 4.1 cannot be explained by
looking at the numerical results. At $\sigma_r = 6.7$ a transition occurs from $D(1)+S(1)$ to $D(1)+S(2)$ and back to $D(1)+S(2)$ with increasing backlash size. This can be caused by measurement errors or is a result of non-linear phenomena which are not covered in the numerical model.

4.2.2 Severity of impacts

Chapter 3.3 gave an overview of the severity of impacts in the numerical model. The metric that defines the severity is the peak-to-peak acceleration. Figure 4.4 shows the peak-to-peak acceleration in the experimental setup as a function of the load ratio and the backlash size. The vertical axes shows the impact severity in which high values represent severe cases of impact. The black dots represents the actual measurements taken.

Figure 4.4: Experimental measurements of the peak-to-peak acceleration metric as a function of the load ratio and the backlash size

The trends in the experimental results agree with the trend in the numerical simulations. The impact severity monotonically increases with increasing force ratio. However, the impact severity seems to have an extremum as function of the backlash size. This confirms the statements of both Krenz and Crowter/Singh which at first sight seemed to contradict.
Chapter 5

Conclusions and Recommendations

5.1 Conclusions

This report extends research conducted by Crowther and Singh[7] on geared systems with backlash under impulsive loading. A numerical model of a four degree-of-freedom system is developed. This model can be seen as a drive line system which is reduced in complexity. The model includes a backlash gap but ignores the effect of friction. The numerical model is used to study the transient dynamics as a result of a reduced drive torque, an engine tip-out. Focus lies on the impacts that occur between the backlash bodies. These impacts have been qualified into combinations of Double Sided and Single Sided Impacts as a function of the static/dynamic loading ratio and the backlash size. Furthermore an experimental setup is designed to verify the numerical results.

The global trend in both numerical and experimental results show that more double sided impacts occur at higher load ratios and smaller backlash gaps. Furthermore, transitions have been observed between Double Sided and Single Sided Impact regions. The results agree with earlier findings and give a more extensive overview of the transition between different regions.

The severity of the impacts is quantified with the peak-to-peak acceleration of the colliding bodies. This metric has been determined with changing static/dynamic load ratios and backlash gap size. Increasing load ratios increase the severity of the impacts. However, there appears to be an extremum in the severity of impacts with increasing backlash size. This could be an explanation for the contradiction found by Krenz (‘clunk severity increases with drive line lash, but not necessarily in a linear manner’) and Crowther/Singh (‘For this particular system the dynamics therefore yield a case where reduced lash is leading to more severe case of clunk(s)). The observed behavior can explain both these statements. The results can be important in the design of engine control strategies to reduce impact response.
5.2 Recommendations

Future research on this topic could address the following issues:

- Include dry friction into the numerical model and match numerical parameters to experimental system parameters.
- Study the physical relevance of the peak-to-peak acceleration as a metric for impact quantification.
- Study additional torque inputs besides engine tip-out.


