Real-time control for a Fuel Cell Hybrid Vehicle with the Equivalent Consumption Minimization Strategy

S.A.K van Loenhout
Reportnumber: DCT 2007.152

Master Thesis

Supervisor:
prof. dr. ir. M. Steinbuch

Coaches:
dr. P.A. Veenhuizen
dr. ir. J.T.B.A. Kessels
dr. ir. T. Hofman

University:
Technische Universiteit Eindhoven (TU/e), The Netherlands
Department of Mechanical Engineering
Section Automotive Engineering Science, Power Trains

Eindhoven, December 2007
**Preface**

This report describes the research I performed to obtain my Master Assignment at the department Mechanical Engineering of the Eindhoven Technical University. The research was performed within a project, which was an initiative of the Dutch Society for Nature and Environment. The distinction of the assignment to be both part of the realization of the mock up for the 2007 AutoRai and performing the research for on-line Energy Management (EM) strategy made this assignment for me very interesting. I would like to thank Niels Scheffer, Martin Leegwater and Bart de Vries for the nice cooperation at the realisation of the mock-up. Further I would like to thank John Kessels and Theo Hofman for their valuable advice and usable comments on Energy Management (EM) strategies in hybrid vehicles. At last I would like to thank Bram Veenhuizen for his input during the discussions concerning the project and the nice cooperation.

December 2007,

Stefan van Loenhout.
# Table of contents

Preface ........................................................................................................................................ 1

Abstract ...................................................................................................................................... 5

Samenvatting .............................................................................................................................. 6

Chapter 1: Introduction .............................................................................................................. 7
  1.1 The project and Master Assignment objective ................................................................. 7
  1.2 Research approach and report outline .............................................................................. 8

Chapter 2: Fuel Cell hybrid, principles and layout ................................................................. 9
  2.1 Power train topology ........................................................................................................ 9
  2.2 Fuel Cell system ............................................................................................................. 11
  2.3 Ultra Capacitor ............................................................................................................... 13
  2.4 Power train component sizes .......................................................................................... 13

Chapter 3: Vehicle modelling .................................................................................................. 15
  3.1 Vehicle ........................................................................................................................... 15
  3.2 Fuel Cell ......................................................................................................................... 17
  3.3 Ultra Capacitor ............................................................................................................... 19
  3.4 Electric Machines ........................................................................................................... 21

Chapter 4: Energy Management for Fuel Cell Hybrid Vehicles .............................................. 22
  4.1 Problem definition .......................................................................................................... 22
  4.2 Optimal control strategies .............................................................................................. 24
  4.3 Real-time control strategies ............................................................................................ 24
  4.3.1 ECMS .......................................................................................................................... 25
  4.4 Evaluation of the equivalent weight factor .................................................................... 27
  4.4.1 Probability function ................................................................................................. 27
  4.4.2 Penalty function ....................................................................................................... 28
  4.4.3 P(I) controller .......................................................................................................... 29
  4.4.4 Active reference value $Q_{\text{ref}}$ ................................................................................ 29
  4.5 ECMS controller model ................................................................................................. 30

Chapter 5: Simulation results ................................................................................................... 32
  5.1 ECMS results with constant equivalence factor ............................................................. 32
  5.2 ECMS results with P controller on equivalence factor .................................................. 36
  5.2.1 Optimization of component sizes ............................................................................ 38
  5.2.2 ECMS under various conditions ............................................................................. 41
  5.2.3 ECMS on different drive cycle ................................................................................ 43
  5.3 ECMS vs. DP ................................................................................................................. 45
  5.4 Results with active reference value $Q_{\text{ref}}$ .................................................................. 48

Chapter 6: Conclusions and recommendations ........................................................................ 50
  6.1 Conclusions .................................................................................................................... 50
  6.2 Recommendations .......................................................................................................... 52
Abstract

During the last couple of years, there is an increased interest in clean, clever and quiet vehicles. The Dutch Society for Nature and Environment wanted to focus attention on the harmful effects of cars on the environment and challenged the department of Mechanical Engineering of Eindhoven University of Technology to contribute to the design of a power train for a sustainable car for the future. This led to the realisation of a mock-up for the “c.mm,n” vehicle, which was presented at the 2007 AutoRai. This assignment deals with an on-line Energy Management (EM) strategy for the “c.mm,n” power train that makes use of the Equivalent Consumption Minimization Strategy (ECMS). The possibilities for implementation are explored by looking at cost prices of the components and sensitivity of the control parameters. At last its performance is compared with an optimal solution, calculated with the Dynamic Programming (DP) algorithm.
Samenvatting

De laatste jaren is de aandacht voor schone, slimme en stille voertuigen steeds meer toegenomen. De Stichting Natuur en Milieu wilde de aandacht trekken voor de schadelijke effecten die auto’s hebben op het milieu en daagde de divisie “Mechanical Engineering” van de Technische Universiteit Eindhoven uit om een bijdrage te leveren aan het ontwerp voor een aandrijving voor een duurzame auto voor de toekomst. Dit heeft geleid tot de totstandkoming van een model voor de “c,mm,n”, welke representeerd is op de AutoRai van 2007. Deze opdracht gaat over een on-line Energie Management (EM) strategie voor de aandrijving van de “c,mm,n”, die gebruikt maakt van de “Equivalent Consumption Minimization Strategy” (ECMS). De mogelijkheden om deze strategie te implementeren zijn onderzocht door te kijken naar de kosten van de componenten en de gevoeligheid van de parameters van de ECMS controller. Tenslotte zijn de prestaties vergeleken met een optimale oplossing die uitgerekend is met behulp van het Dynamisch Programeren (DP) algoritme.
Chapter 1: Introduction

1.1 The project and Master Assignment objective

The foundation for Nature and Environment (“Stichting Natuur en Milieu”, SNM) was the initiator for the project “Car of the Future”. The project started in August 2005, by challenging the three technical universities of the Netherlands (Eindhoven, Delft en Twente) to make a sustainable car for the future. This as an answer to the 2005 AutoRai, where very little attention was drawn by cars that are developed to be less harmful to the environment. The upcoming concerns about global warming through CO_2 emissions due to cars and the attention SNM wanted to focus on this problem caused them to make a statement to the car industry. SNM wanted them to show an example of a clean, clever and quiet car so as to encourage the car manufacturers to quickly start mass production of “green” cars.

After a short inquiry at the three universities (3TU), it proved to be better to combine the efforts of the three universities instead of competing each other.

The “car of the future” project involves the design of the exterior, interior, suspension system and power train of a future car within the context of the community in 2020. The mechanical engineering division of the Eindhoven Technical University (TU/e) was asked to work out the whole suspension and power train system of what is called, the “c,mm,n” vehicle.

One student from the TU/e was responsible for the active suspension system of the car. He developed a system, which was able to keep the vehicle levelled under any circumstances without using additional energy. The choices for the drive train were made by two students, which lead to two different power train configurations that can work on hydrogen. One power train consist of a traditional combustion engine [19], and the other one is a hybrid drive train carrying a Fuel Cell (FC) with an Ultra Capacitor (UC) as secondary power source [21]. This report is a continuation of the work preformed by this former student, which made the topology choices for the FC power train and optimized the power train for size and energy consumption using Dynamic Programming (DP).

To be able to present the FC hybrid drive train on the 2007 AutoRai in a proper way, the decision was made to make a mock up of the drive train. Therefore the assignment consists of two part. The first part was to realise the mock up for the AutoRai. Therefore a design for the power train compartment in the vehicle had to be made, and sponsors had to be found to realise the mock-up with the actual components. Figure 1.1 shows the result of the suspension and power train model presented on the AutoRai.

The second part of the assignment focuses on the energy management (EM) of the FC hybrid power train in the “c,mm,n” vehicle. The global energy management solution from DP is not suitable for real-time use. Therefore an (EM) strategy which can be used for real-time implementation is investigated within this assignment. This report deals with the second part of the assignment.

The goal of this assignment can be stated as:

“Investigate and explore the possibilities of an Energy Management (EM) strategy which can be used for on-line vehicle implementation. Compare its performance with the global EM solution from Dynamic Programming (DP)”.
1.2 Research approach and report outline

To obtain understanding of the hybrid technology, the layout of the power train and the principles of its components are described in Chapter 2. The simulation environment is explained in Chapter 3. The different simulation models are exposed which were implemented in the overall vehicle model. Chapter 4 contains the study of the different Energy Management (EM) strategies that are discussed in many literature on hybrid power train control. Both the off-line as well as the on-line EM strategies are investigated to be able to design a real-time strategy suitable for the simulation environment. Chapter 5 presents the simulation results obtained during the investigation of the EM strategy. The need for a controller on the State-Of-Charge (SOC) of the Ultra Capacitor (UC) is explained in Section 5.1. The results achieved with this controller are presented in Section 5.2, where a distinction is made for the optimal component sizes on costs and fuel economy and the sensitivity of the EM strategy under various driving conditions. Section 5.3 is dedicated to the performance of the strategy in contrast with a non-hybrid Fuel Cell (FC) vehicle and the optimal solution for EM calculated with Dynamic Programming (DP), where Section 5.4 shows an proposal for an active reference value for the controller. Finally in Chapter 6, the conclusions from the research are presented as well as recommendations for further research on implementing the EM strategy in a real world environment.
Chapter 2: Fuel Cell hybrid, principles and layout

In Chapter 1, the outline of this thesis is discussed. Based upon the given criteria from the initiators of the project “Car in the Future”[23], choices for the different components were made, and a final topology for the power train was designed. Via the “methodical design method”, this has been worked out in [21]. This chapter provides a short review of the different choices that had to be made concerning the drive train topology and provides insight in the principles of the different drive train components.

2.1 Power train topology

The “c,mm,n” vehicle is a Fuel Cell Hybrid Vehicle (FCHV), which is powered by electric in-wheel motors running on electricity generated by the Fuel Cell (FC) stack. The FC uses hydrogen as its energy source. The “c,mm,n” car is equipped with a system that combines a FC stack and Ultra Capacitor (UC) with onboard high pressure hydrogen tanks. The FC stack serves as the main power source, while the UC contributes its storage capabilities as supplementary power source to deliver ample drive power to the electric motors.

![Figure 2.1: “c,mm,n” vehicle power train layout](image)

In [21] hydrogen was chosen as the fuel of the future. This hydrogen is converted into electricity using a FC. A FC for mobile applications has to meet certain criteria. Besides the technical criteria, also criteria about sustainability will have to be taken into account. Looking at this criteria, a good choice for a mobile application is a FC system with a dry electrolyte (safety) and a low operating temperature (quick start-up and power demand response). The best option is the PEM FC (Polymer Electrolyte Membrane Fuel Cell), which is very suitable for these applications, since it also features a dry electrolyte, but mainly because it operates at a relatively low temperature, between 335 and 375 [K]. The PEM FC is also the most common used type of FC in mobile applications nowadays.
Electric in-wheel motors in every wheel, are chosen because of their benefits and the desire to power all the wheels. The traction forces are divided over all four wheel, resulting in less torque per wheel. The vehicle stability systems will have the possibility of distributing the power to the wheel with the most grip in slippery conditions. And in general, four wheel drive configurations are known to give better vehicle dynamics. It is also of benefit for the energy recuperation, reclaiming power at all four wheels.

To make energy recuperation possible, a UC is added as a secondary power source. An UC has the highest specific power density of 1500 [W/kg], and has a high (dis)charge rate. Also the expected life time is very long, which makes this technology favourable over batteries.

Figure 2.2 shows some driving modes, which are possible due to the hybridization of the power train. In the Chapter 3, the Energy Management (EM) within the power train is explained in more detail.

![UC assists FC for better performance at accelerations](diagram1)

![During regenerative braking, the UC stores the released energy by the electric motors](diagram2)

![At constant driving, the drive power can be delivered by the FC only. Excess power can be stored in the UC.](diagram3)

![At standstill, the FC delivers the power demanded by the auxiliaries. The UC can be charged as well.](diagram4)

**Figure 2.2: Energy flow for FCHV**

The energy efficiency of vehicles can be separated in two parts. First, one has to consider the efficiency of the fuel manufacturing plant (well-to-tank efficiency) and second, there is the efficiency of the vehicle itself (tank-to-wheel efficiency). Altogether, the overall efficiency (well-to-wheel efficiency) is equal to the product of both efficiencies. Table 2.1 shows the efficiencies for three vehicle configurations: the traditional gasoline car, the Hybrid Electrical Vehicle (HEV), and the FCHV with hydrogen.
Table 2.1: Overall fuel efficiency [13]

<table>
<thead>
<tr>
<th></th>
<th>Well-to-tank efficiency (%)</th>
<th>Tank-to-wheel efficiency (%)</th>
<th>Walk-to-wheel efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional gasoline car</td>
<td>88</td>
<td>16</td>
<td>14%</td>
</tr>
<tr>
<td>Hybrid electric vehicle</td>
<td>88</td>
<td>37</td>
<td>32%</td>
</tr>
<tr>
<td>Fuel Cell hybrid vehicle</td>
<td>58</td>
<td>50</td>
<td>29%</td>
</tr>
</tbody>
</table>

The table shows an excellent well-to-tank efficiency for gasoline through the fully optimized manufacturing process. In spite of the higher tank-to-wheel efficiency of the FCHV, the HEV currently dominates the hydrogen technology. In the near future, manufacturers like Toyota, expect the tank-to-wheel efficiency to improve 10%. Together with an improved well-to-tank efficiency, this will put the FCHV in a leading position.

In the next sections the working principles of the PEM FC stack and UC within the FCHV are provided in more detail.

2.2 Fuel Cell system

The construction of a PEM FC consists of various parts. The anode and cathode have small canals in which the hydrogen and air are brought into contact with the catalyst to facilitate the splitting reaction. In the middle of the construction, the electrolyte (the proton exchange membrane) is placed. A schematic picture of the parts of the PEM FC stack can be seen in figure 2.3a.

Figure 2.3: (a) Arrangement of a PEM FC stack. White fields represent the proton exchange membrane, grey fields are the flow fields of reactant gases, black field are bipolar plates. (b) Principles of operation of a fuel cell [2],[9]
Single cells are separated by flow fields that supply reactant gas to each electrode. Flow fields are arranged in bipolar plates, which also have to conduct electricity to the external circuit and allow the water flow to remove heat generated by the reaction. At the anode of the FC, $H_2$ is split in two protons and two electrons with the help of the platinum catalyst:

$$H_2 \rightarrow 2H^+ + 2e^- \quad (2.1)$$

The catalyst is a special material that facilitates the reaction of oxygen and hydrogen. It is usually made of platinum powder very thinly coated onto carbon paper or cloth. The catalyst is rough and porous so that the maximum surface area of the platinum can be exposed to the hydrogen or oxygen. After this splitting reaction of hydrogen the $H^+$ ions diffuse through the electrolyte that is only permeable for hydrogen and then go to the cathode. The electrons that exist during the splitting reaction of hydrogen diffuse through the anode and are going to the external electrical circuit and afterwards to the cathode, figure 2.3 b. This current $I$, can be used to power an external load. On the cathode side of the FC, oxygen gas ($O_2$) is being forced through the catalyst, where it forms two oxygen ions. Each of these ions has a strong negative charge. This negative charge attracts the two protons through the membrane, where they combine with an oxygen atom and two of the electrons from the external circuit to form a water molecule ($H_2O$):

$$2H^+ + 2e^- + \frac{1}{2}O_2 \rightarrow H_2O \quad (2.2)$$

The overall reaction becomes:

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O \quad (2.3)$$

A FC stack needs to be integrated with other components to form a FC system able to power a vehicle. Figure 2.4 shows a hybrid electric FC system able to power a vehicle. One can recognize the air, hydrogen and cooling system.

![Figure 2.4: FCHV power train system, with air and cooling circuit for fuel cell [2],[9]](image-url)
2.3 Ultra Capacitor

Ultra Capacitors (UC) store energy in the electric field of an electrochemical double layer. Compared to batteries, ultra capacitors show a very high specific power 500-2500 [W/kg] and a low specific energy of 0.2-6 [Wh/kg], depending on the used materials. This makes them only suitable for short term energy storage systems in FCHV applications, to deliver short high power burst. They are being developed for power assists during acceleration as well as for the recovery of braking energy due to their ability to accept high instantaneous charge rates. In the configuration of a FCHV, they make it possible to downsize the FC to the average power demand depending on the energy content of the UC. An UC differs from conventional capacitors both in the materials of which it is made and in the physical processes involved. In an UC the dielectric is an ion-conducting electrolyte interposed between conducting electrodes. The energy is stored by the charge separation taking place in the layers that separate the electrolyte and the electrodes. Figure 2.5 shows a schematic representation of an UC.

![Figure 2.5: Schematic of an Ultra Capacitor](image)

Since the applied voltage is limited to a few volts, the storage capacity is increased, by increasing the surface and decreasing the thickness of the electrolyte. The electrodes are therefore made of porous materials, which increase the surface due to its fine structure. The electrodes are separated by an insulating ion-conduction membrane, called separator. The most common materials for automotive applications are carbon-based cells with polymer electrolyte, which seem to offer the best performance at the lowest costs.

2.4 Power train component sizes

To get a good impression of what the maximum power necessary for the vehicle would be, the “c,mm,n” vehicle was simulated in the SFTP-US06 Highway cycle. With the specifications of the “c,mm,n” [23], 50 [kW] was the maximum power required at the wheels. The 0 to 100 km/h acceleration of the vehicle had to be comparable with cars that are nowadays market leader in the C segment, which was estimated on 12 [s]. Using:

$$ t_0 \approx \frac{v_0^2 m_u}{P_{\text{max}}}, \quad (2.4) $$

an acceleration of 13 [s] is realizable. This is assumed to be acceptable [21]. Together with a maximum efficiency of 90% for the electric motors, 98% efficiency for converters and a
margin for electric loads for auxiliaries, a power train with a 60 [kW] electric output should be enough. An efficiency of 98% for the converters is hardly achievable, but used to make a good comparison with earlier simulations [21]. The average power demand over a drive cycle is often considerably lower than the maximum power demand. For the NEDC and the SFTP US60 highway cycle the average power demand is 5.7 [kW] and 16.4 [kW] respectively. To be able to charge the UC under any circumstance the output of the FC has to be chosen higher. A validation of these dimensions in [21] for the most fuel efficient configuration, resulted in a 50 [kW] FC with a 30 [kW] UC.
Chapter 3: Vehicle modelling

In this chapter, the main simulation models within the power train of the “c,mm,n” vehicle are described. For simulation and comparison aims it is convenient that various drive train configurations can be modelled easily. Therefore the QSS-toolbox (Quasi Static Simulation Toolbox) is chosen for simulation purpose [10]. The QSS-toolbox is a collection of Simulink blocks and the appropriate parameter files that can be run in a Matlab/Simulink environment. Complex power train structures can be built with basic blocks. The QSS-Toolbox makes it possible for power train systems to be designed quickly and in a flexible manner, to easily calculate the fuel consumption of such systems. Due to the extremely short CPU time it requires, it can be used for design exploration. The models consist of static equations and efficiency maps of the components. Figure 3.1 shows the top layer of the total power train model within the QSS-Toolbox, which uses a so called ‘backward power flow’.

Figure 3.1: Drive train model in QSS-Toolbox

Looking at figure 3.1 from left to right one can recognize the total vehicle model back to front. The model starts with the imposed drive cycle, which determines the drive power demand. The vehicle model calculates the corresponding rotational wheel speeds and accelerations, which are offered to the in-wheel motor. Torque is divided by 4 (number of in-wheel motors) and the input power is multiplied by the same value to get the right power in the Energy Management (EM) controller. Here the power split is determined. The block “auxiliaries” adds the auxiliaries power to the power demand of the Fuel Cell (FC). “Energy Source” at last calculated the energy demand over the whole cycle. The next sections will be used to explain the main individual models.

3.1 Vehicle

The backward power flow method uses pre-defined drive cycles to determine vehicle speed and acceleration. The parameters $v$ and $dv$ are used by the vehicle model to calculate the parameters needed for the electric motors. Therefore the vehicle parameters, which are presented in table 3.1, need to be defined. These parameters were determined in [21] and for comparability of the results used within this investigation as well.
Vehicle mass (no drive train component) $m_v$: 650 [kg]

Frontal area $A_f$: 2.1 [m$^2$]

Roll drag $c_r$: 0.01 [-]

Air drag $c_d$: 0.23 [-]

Wheel radius $R_w$: 0.15 [m]

Auxiliaries $P_{aux}$: 1000 [W]

### Table 3.1: Vehicle parameters [21]

The torque required to complete the loaded drive cycle are calculated, using the following formulas [9]:

$$T(t) = F_i(t)R_w$$  \hspace{1cm} (3.1)

$R_w$ is the wheel radius and $F_i(t)$ is the total force of the vehicle calculated using:

$$F_i(t) = F_{air}(t) + F_{acc}(t) + F_r(t)$$  \hspace{1cm} (3.2)

With $F_{air}$ the force applied by air drag:

$$F_{air}(t) = \frac{1}{2} \rho_{air} A_f c_d v^2(t)$$  \hspace{1cm} (3.3)

Where $\rho_{air}$ is the air density [kg/m$^3$], $A_f$ the frontal area of the car [m$^2$], $c_d$ the air drag [-] and $v$ the vehicle speed [m/s].

In (3.2), $F_{acc}(t)$ are the inertia forces during acceleration:

$$F_{acc}(t) = m_v (1 + c F_w) \frac{d}{dt} v(t)$$  \hspace{1cm} (3.4)

Where $c F_w$ is the contribution of the inertia of the wheels (and in-wheel motors)[%] and $m_v$ the vehicle mass [kg].

$F_r(t)$ is the drag force from roll resistance:

$$F_r(t) = c_r m_v g$$  \hspace{1cm} (3.5)

Where $c_r$ is the roll drag [-] and $g$ is gravity constant [m/s$^2$].

The auxiliaries power $P_{aux}$ is assumed to be constant on an estimated level of 1000 [W]. This is a reasonable level according [12].
3.2 Fuel Cell

The behaviour of a single cell is characterized in terms of cell voltage $V_{FC}(t)$ and current density $i_{FC}(t)$. This current density has a maximum 6000 [A/m$^2$] in the QSS-Toolbox model. Figure 3.2 shows the look-up table which presents the FC static dependency between the cell voltage $V_{FC}(t)$ and $i_{FC}(t)$. From this look-up table, using the current density $i_{FC}$ as input, the cell voltage $V_{FC}(t)$ can be found.

![Figure 3.2: QSS PEM FC model.](image)

The main FC parameters are listed in table 3.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells $N_{FC}$</td>
<td>6.22 [cell/kW]</td>
</tr>
<tr>
<td>Mass density $\rho_{ FC}$</td>
<td>250 [W/kg]</td>
</tr>
<tr>
<td>Theoretical cell voltage $V_{th}$</td>
<td>1.16 [V]</td>
</tr>
<tr>
<td>Fuel Cell size $A_{FC}$</td>
<td>0.05 [m$^2$]</td>
</tr>
<tr>
<td>Air compression ratio $\Pi_c$</td>
<td>1.3 [-]</td>
</tr>
<tr>
<td>Air compressor efficiency $\eta_c$</td>
<td>0.7 [-]</td>
</tr>
<tr>
<td>E-motor for air compressor efficiency $\eta_{EM}$</td>
<td>0.75 [-]</td>
</tr>
<tr>
<td>Air to fuel ratio $I_{air}$</td>
<td>1.5 [-]</td>
</tr>
<tr>
<td>Estimated cost price $CP_{FC}$</td>
<td>1000 [€/kW]</td>
</tr>
</tbody>
</table>

Table 3.2: Fuel Cell parameters [21]

The current density $i_{FC}(t)$ inside the fuel cell is calculated using:

$$i_{FC}(t) = \frac{P_{\text{ Intern}}(t)}{V_{FC}(t)A_{FC}}$$  \hspace{1cm} (3.6)

With $A_{FC}$ the total active surface area and $P_{\text{ Intern}}$ calculated from (3.7). Since, the voltage in the denominator is a function of the FC current this results in a implicit equation. Therefore an additional delay is introduced in the model, which breaks this implicit loop.

$$P_{\text{ Intern}}(t) = P_{\text{ ask}}(t) \cdot \frac{1}{0.98} + P_{\text{ comp}}(t) + P_{\text{ aux}}(t)$$  \hspace{1cm} (3.7)

$P_{\text{ ask}}$ is the power asked at the FC for that time step $P_{\text{ ask}}(t)$, $P_{\text{ comp}}$ is the power demand of the air compressor and $P_{\text{ aux}}$ is the power demand for other FC auxiliaries. The Value of $1/0.98$ is added to $P_{\text{ ask}}$ to include the losses of the FC converter. The cell voltage $V_{FC}(t)$ multiplied by the number of cells $N_{FC}$ gives the total FC voltage.
\[ V_{FC, tot}(t) = V_{FC}(t)N_{FC} \]  

(3.8)

The cell voltage is divided by the theoretical cell voltage, resulting in the efficiency of the FC.

\[ \eta_{FC}(t) = \frac{V_{FC}(t)}{V_{th}} \]  

(3.9)

This efficiency is used to calculate the power of the FC, which is the equivalent for the fuel consumption.

\[ P_{FC}(t) = \frac{P_{\text{Intern}}(t)}{\eta_{FC}(t)} \]  

(3.10)

The power consumption of the air compressor \( P_{\text{comp}} \) is used to calculate the compressor losses, necessary for (3.7), using:

\[ P_{\text{comp}}(t) = \frac{P_{FC}(t)}{C_{\text{comp}}} \]  

(3.11)

With \( C_{\text{comp}} \) the constant compressor losses [-], calculated by:

\[ C_{\text{comp}} = \frac{1}{2} \frac{21}{100} \frac{LHV_{H_2} M_{air} I_{air} \Delta h_{air}}{\eta_{EM}} \]  

(3.12)

The factor \( \frac{1}{2} \) comes from the ratio \( O_2 : H_2 \) in the reaction, and the factor 21/100 from the percentage of oxygen in the air. \( LHV_{H_2} \) is the lower heating value of hydrogen, \( M_{air} \) is the molecular weight, \( I_{air} \) is the air-hydrogen ratio, \( \eta_{EM} \) is the compressors electromotor efficiency and the enthalpy difference \( \Delta h_{air} \) is calculated using:

\[ \Delta h_{air} = C_{p_{air}} \frac{T_{air}}{\eta_{c}} \left( \frac{\kappa_{air} - 1}{\kappa_{air}} \right) - 1 \]  

(3.13)

In this formula, \( C_{p_{air}} \) is the specific heat of air, \( \kappa_{air} \) is the isentropic efficiency of air and \( T_{air} \) the air temperature. \( \eta_{c} \) is the air compression efficiency and \( \Pi_{c} \) the air compression ratio.

Figure 3.3 shows the output power and efficiency of a 40 [kW] FC stack as a function of the input power of the FC. These are the result of a simulation of a 40 [kW] FC on the NEDC. The small differences for multiple values are caused by the interpolation in the look-up table for the FC model within the QSS-Toolbox.
3.3 Ultra Capacitor

The input parameter for the Ultra Capacitor (UC) model is the required or delivered power $P_{UC}(t)$. The output parameters are the voltage $U_{UC}(t)$ and the charge $Q_{UC}(t)$. A simplified model of UC is shown in figure 3.4. It consists of a capacitor and a resistor in series. The simulation model in the QSS-Toolbox is based on a physical model which is derived from this equivalent model of an UC.

![Electric circuit of a Ultra Capacitor](image)

Table 3.3 presents the UC parameters. For maximum charge $Q_{\text{max}}$ a value of 90% of the maximum charge is chosen. This is a safety margin to prevent the UC to be overcharged.
Table 3.3: Ultra capacitor parameters [21]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity $C_{UC}$</td>
<td>5000 [F]</td>
</tr>
<tr>
<td>Resistance $R_{UC}$</td>
<td>0.4e-3 [ohm]</td>
</tr>
<tr>
<td>Maximum voltage $V_{max}$</td>
<td>2.5 [V]</td>
</tr>
<tr>
<td>Maximum / Minimum charge $Q_{max}/Q_{min}$</td>
<td>90 / 50 [%]</td>
</tr>
<tr>
<td>Energy density $\rho_{E,UC}$</td>
<td>5 [Wh/kg]</td>
</tr>
<tr>
<td>Power density $\rho_{P,UC}$</td>
<td>1500 [W/kg]</td>
</tr>
<tr>
<td>Estimated cost price $CP_{UC}$</td>
<td>100 [€/cell]</td>
</tr>
<tr>
<td>Subsystem mass $m_{subsystem}$</td>
<td>30 [kg]</td>
</tr>
</tbody>
</table>

Equation (3.14) forms the basis of the simulation model of the UC.

$$I_{SC}(t) = \frac{\left( -\frac{Q_{UC}(t)}{C_{UC}} + \sqrt{\left( \frac{Q_{UC}(t)}{C_{UC}} \right)^2 - 4 \cdot R_{UC} \cdot P_{UC}(t)} \right)}{2 \cdot R_{UC}}$$  \hspace{1cm} (3.14)

In this equation, the current is calculated with two constant and two variables. The constants are the capacity $C_{UC}$ and the internal resistance $R_{UC}$ . The two variables are the required or delivered power $P_{UC}(t)$ and the actual charge $Q_{UC}(t)$ . $P_{UC}(t)$ is calculated using:

$$P_{UC}(t) = P_{\text{ask}}(t) \cdot \frac{1}{0.98} + P_{UC,\text{loss}} \text{, if } P_{UC} > 0$$
$$P_{UC}(t) = P_{\text{ask}}(t) \cdot 0.98 + P_{UC,\text{loss}} \text{, if } P_{UC} < 0$$  \hspace{1cm} (3.15)

In order to include the losses if the UC converter, multiplying $P_{\text{ask}}(t)$ with a constant is necessary.

The power losses $P_{UC,\text{loss}}(t)$ are calculated with:

$$P_{UC,\text{loss}}(t) = R_{UC} \cdot I_{UC}^2(t)$$  \hspace{1cm} (3.16)

For high voltage applications such a Fuel Cell Hybrid Vehicles, a series chain of UCs must be used to avoid exceeding the working voltage of the individual UC. But a series connection reduces the total capacity of the chain. For a series chain of $N_{UC,\text{ser}}$ equal value UCs the capacity is calculated as follows:

$$C_{tot,\text{ser}} = \frac{C_{UC}}{N_{UC,\text{ser}}}$$  \hspace{1cm} (3.17)

Where $C_{tot,\text{ser}}$ is the capacity of the chain and $C_{UC}$ is the capacity of the individual UC. At the same time, the internal resistance of the chain $R_{tot,\text{ser}}$ is increased to:

$$R_{tot,\text{ser}} = R_{UC} \cdot N_{UC,\text{ser}}$$  \hspace{1cm} (3.18)

Where $R_{UC}$ is the internal resistance of the individual UC.
Higher capacitances can be achieved by using parallel UCs. In this case the capacity of a module of $N_{UC,par}$ parallel UCs $C_{tot,par}$ is given by:

$$C_{tot,par} = C_{tot,ser} \cdot N_{UC,par}$$ (3.19)

At the same time the resistance of a module is reduced and is calculated as follows:

$$R_{tot,par} = \frac{R_{tot,ser}}{N_{UC,par}}$$ (3.20)

The energy content of an UC is given by:

$$E_{UC}(t) = \frac{1}{2} C_{UC}(t) \cdot V_{UC}^2(t)$$ (3.21)

Where $V_{UC}$ is the voltage over the ultra capacitor calculated using:

$$V_{UV}(t) = \frac{Q_{UC}(t)}{C_{UC}(t)}$$ (3.22)

### 3.4 Electric Machines

The torque required and the speed at the wheels as prescribed by the drive cycle are used as inputs for the model of the electric machines. The relationship between the power required at the shaft and the input power required can be calculated without a detailed model of the electric motor when a stationary map of the machine efficiency $\eta_{EM}$ as a function of the input variables $T_{EM}(t)$ and $\omega_{EM}(t)$ is available. A two quadrant efficiency map for a typical traction motor is defined in the standard QSS-TB package.

For comparability purpose the standard electric motor is replaced by the Auxilec Thomson 32 [kW] (continuous), permanent magnet motor/controller. A look-up table is used, adapted from Advisor, which holds the input power as a function of $T_{EM}(t)$ and $\omega_{EM}(t)$. This look-up table is implemented in the standard QSS model instead of the efficiency map, which requires a small adaptation of the model. The motor speed map is divided by a constant to match the maximum rpm of the wheels, and the torque is multiplied with the same constant to keep the power output equal.

From these inputs, the electric power demand of the wheel motors is given as outputs. The positive values of the power demand are divided by the ac/dc converter, and the negative values are multiplied by the same converter efficiency.
Chapter 4: Energy Management for Fuel Cell Hybrid Vehicles

Energy Management (EM) strategies for Fuel Cell Hybrid Vehicles (FCHV) are algorithms that choose at each time the power-split between the primary, Fuel Cell (FC), and secondary power source, in this case an Ultra Capacitor (UC). The strategies can be divided into two different types, i.e., off-line and on-line EM strategies. The off-line strategies are related to simulation purpose, since it is not the main goal to derive real-time solutions, yet global solutions. These EM strategies are often called: Optimal control strategies.

The on-line strategies are able to manage the power train for real-time control. This type of strategy chooses the optimal operating point at each time. These are the “Real-time control strategies”. Both types consist of different approaches to calculate the power split.

The Equivalent Consumption Minimization Strategy (ECMS) is used in Chapter 5 to investigate the performance of the “c,mm,n” vehicle for a real-time control strategy. In the end of this chapter a proposal for the on-line implementation of the ECMS is done.

4.1 Problem definition

The power train of the “c,mm,n” vehicle features a 50 [kW] PEM Fuel Cell stack (FC) and a Ultra Capacitor module (UC) made of 5000 [F] UC’s, capable of producing 30 [kW] output power. The UC module consists of 92 series connected UC’s with a total capacity of 54.34 [F]. The main goal of the EM strategy is to improve the vehicle’s fuel economy. This is done by using the most fuel efficient power split between the two power sources. Within this strategy different driving modes can be identified:

- drive power is delivered by FC, denoted with $P_{FC}$,
- drive power is delivered by UC, denoted with $P_{UC}$,
- drive power is delivered by both FC and UC,
- drive power is regenerated by UC,
- drive power is regenerated by UC, while FC provides extra charge power for driving,
- FC delivers power for auxiliaries (at standstill),
- FC charges UC and delivers power for auxiliaries (at standstill).

The power train topology corresponds to a series hybrid architecture. Both the FC and the UC contribute to the supply of the vehicle power. Figure 3.1 shows the power train layout, where the different blocks represent the different components present. The auxiliaries represent all the electric component that are not related to the drive train, i.e., the air conditioning, radio, navigation, etc. The DC-link is necessary to connect the drive-train components and to make them work on the same voltage, since the voltage of the FC and UC are not constant.
The motor power \( P_{EM} \) is assumed to be positive during traction phase and negative during braking phase. The FC power \( P_{FC} \) is necessary positive and the UC power \( P_{UC} \) is positive during discharge and negative during charge. The powers relation is:

\[
P_{EM}(t) = P_{FC}(t) + P_{UC}(t)
\] (4.1)

The power distribution is optimized to minimize the overall hydrogen consumption over a given driving cycle. The objective becomes:

\[
\text{Min} \sum_{t=0}^{T} P_{H2}(t) \Delta t
\] (4.2)

The variable \( P_{H2}(t) \) is the hydrogen power output of the FC \( P_{FC}(t) \) at time \( t \).

The hydrogen power \( P_{H2} \) is the product of the hydrogen mass flow rate \( m_f \) and the lower heating value of hydrogen \( \text{LHV}_H2 \).

The operating range of the electric motor, FC and UC are limited due to power specifications. Therefore, inequality constraints are introduces to limit the minimum and maximum power flow through these components. Also the charge \( Q \) of the UC is limited.

\[
0 \leq P_{FC}(t) \leq P_{FC_{\text{max}}}(t)
\]

\[
P_{EM_{\text{min}}}(t) \leq P_{EM}(t) \leq P_{EM_{\text{max}}}(t)
\]

\[
P_{UC_{\text{min}}}(t) \leq P_{UC}(t) \leq P_{UC_{\text{max}}}(t)
\]

\[
Q_{\text{min}} \leq Q(t) \leq Q_{\text{max}}
\] (4.3)

Not only the overall fuel consumption of the vehicle is important, but the charge sustainability of the vehicle is also an important constraint. Since, the UC is depleted at the end of the cycle, the energy is added to the power train instead of generated within the system.

\[
\Delta SOC \big|_{t_f} = \frac{Q(t_f) - Q(0)}{Q(0)} = 0
\] (4.4)

Where the charge \( Q(t) \), representing the state of the UC. When this constraint is satisfied no energy is added to the drive train and a comparison with other vehicles can be made.
4.2 Optimal control strategies

In literature several algorithms for the global optimization of the Energy Management (EM) systems are described. They all make use of the following techniques to optimise the fuel consumption:

- Dynamic Programming (DP) [5,14,21]
- Optimal control[3,6,7,9]

All of the results are based on a priori knowledge of the future driving conditions, as provided by scheduled drive cycles. This is done to find the most fuel efficient EM strategy over a certain drive cycle, considering all possibilities of (dis)charging the secondary power source. Therefore these algorithms are not suitable for real-time control, but they can be used as a benchmark for real time control strategies.

The DP algorithm was used in [21] to calculate the minimum fuel use and the optimal component sizes of the vehicle on different drive cycles. These simulation results are used to evaluate the performance of the real-time strategy to see whether a real-time strategy can be as efficient as a off-line calculated optimal solution.

As stated before, DP calculates an optimal solution with a priori knowledge of the future driving conditions. DP calculates at each time step the total power request of the drive cycle. This total power request is stored in a map, together with all possible charge levels calculated for each time step in the drive cycle. Since each charge level depends on earlier changes of charge decisions, a starting point has to be chosen. The change in charge of the UC is a result of the power delivered or retrieved by the UC. With both $P_{EM}$ and $P_{UC}$ known, automatically $P_{FC}$ is known.

For every second and every possible charge level of the UC, the minimum cost is stored in another map. At the end, the path of the lowest fuel cost is calculated back. In this way the optimal solution can be calculated by following the path with the lowest FC power and so the lowest possible fuel consumption. Results of DP for the “c,mm,n” car are presented in Chapter 5. To determine the optimal power train component sizes for “c,mm,n”, different configurations are simulated with DP. From here, the most fuel efficient configuration is defined. For more detailed information about the DP routine the reader is referred to [14,21].

4.3 Real-time control strategies

The development of a real-time Energy Management (EM) control strategy, based on an optimization in real-time, has been subject of investigation in literature.

In this “local optimization” approach two main difficulties must be encountered:

- no or very limited knowledge of the future driving conditions is available during actual operation, and
- the charge of the UC must be sustained by breaking or by charging with the FC.

The core of the local optimization strategy is the definition of a cost function that is to be minimized, which depends only upon the systems variables at that instant time. Not only the
fuel energy use, but also the variation in stored reversible energy has to be taken onto account within this cost function. There are different approaches proposed which deal with this problem in several ways:

- rule-based control strategies that are based on ‘if-then’ type of control rules. [9,15];
- introduce a tuning parameter, that effects the optimization, adjusted according to SOC deviation by PID controller. [8];
- cost function as sum of all losses in reversible and thermal paths. [22], and
- introduce equivalence factor: cost function as sum of the fuel consumption and an equivalent fuel consumption (ECMS). [11,13,17,20]

This Equivalent Consumption Minimization Strategy (ECMS) is chosen to evaluate real-time EM behaviour and is further described in this section, since this EM strategy is rather straightforward and can be easily implemented into the vehicle model. This EM strategy is implemented in the vehicle model so a comparison of this real-time strategy with the optimal DP routine can be made. The results of the simulations can be seen in Chapter 5.

### 4.3.1 ECMS

The ECMS strategy makes use of a cost function that evaluates the sum of the fuel consumption and the equivalent fuel consumption due to the State-Of-Charge (SOC) variation in the Ultra Capacitor (UC).

From 4.1 could be seen that the power distribution in the vehicle can be described as follows:

\[
P_{\text{EM}}(t) = P_{\text{FC}}(t) + P_{\text{UC}}(t)
\]

with \(P_{\text{UC}}\) negative at charge.

The optimal EM strategy follows from the general optimization problem:

Minimize \[\sum_{0}^{\tau_f} P_{H2}(t) \Delta t\]  \hspace{1cm} (4.6)

Subject to

\[
0 \leq P_{\text{FC}}(t) \leq P_{\text{FC max}}(t)
\]

\[
P_{\text{EM min}}(t) \leq P_{\text{EM}}(t) \leq P_{\text{EM max}}(t)
\]

\[
P_{\text{UC min}}(t) \leq P_{\text{UC}}(t) \leq P_{\text{UC max}}(t)
\]

\[
Q_{\text{min}} \leq Q(t) \leq Q_{\text{max}}
\]

\[
\Delta SOC\bigg|_{\tau_f} = \frac{Q(t_f) - Q(0)}{Q(0)} = 0
\]  \hspace{1cm} (4.7)
The variable $P_{H2}(t)$ is the hydrogen power output of the Fuel Cell (FC) $P_{FC}(t)$ at time $t$.

The hydrogen power $P_{H2}$ is the product of the hydrogen mass flow rate $m$ and the lower heating value of the hydrogen LHV$_{H2}$.

This global minimization problem needs knowledge of driving cycle a priori. Thus real-time control cannot readily be implemented.

The ECMS proposes to replace the global criterion by a local one, which reduces the problem to a minimization of an equivalent fuel consumption at any instance. For each time $t$ with a time step $\Delta t$, ECMS makes use of an equivalent fuel cost function $J(t)$. This equivalent fuel cost function uses an electric-energy-to-fuel-conversion-weight-factor, or equivalent weight factor $s(t)$.

$J(t) = P_{H2}(t) + s(t)P_{UC}(t)$ \hspace{1cm} (4.8)

$s(t)$ facilitates the conversion of the electrical power flow into a chemical power flow, taking into account the efficiency of the power plant.

The optimal momentary power set-point $P_{UC}^o(t)$ for the secondary power source is the power, which minimizes equation (4.8), giving a certain $s(t)$.

$P_{UC}^o(t) = \arg \min_{P_{UC}(t)} (J(t), s(t))$ \hspace{1cm} (4.9)

Evaluation of the equivalent weight factor is basically the core of the ECMS strategy, because it influences the system as follows.

$s(t)$ too large: use of UC is penalized $\Rightarrow$ fuel consumption increase.
$s(t)$ too small: use of UC is favoured $\Rightarrow$ UC is depleted.

The evaluation of this weight factor is discussed later on. The first part of the investigation focuses on a constant equivalence factor for a specific drive cycle. In [13] can be read that there exists a unique solution for this equivalence factor $s$ on a specific drive cycle where the energy for discharge equals the energy for charge in the UC. That strategy should mimic the solution for DP, when the operating boundaries of the UC are never crossed. Section 4.4 discusses the situation where the boundaries are reach due to limited capacity.

First a constant equivalence factor $s$ for the ECMS strategy is introduces. The idea is to calculate the optimal cost function $J(t)$ for different values of the equivalence factor $s$, and plot those values of $s$ against the $\Delta SOC$. The value of $s$ which yields a $\Delta SOC \approx 0$ is an equivalence factor for which the solution complies with the problem formulation (4.6) and (4.7).

Therefore the control variable $P_{UC}$ is defined as:

$P_{UC} = [P_{UC,\min} : P_{UC,\max}]$ \hspace{1cm} (4.10)

This vector, which contains all possibilities for $P_{UC}$, is offered to the controller, which calculates the actual value of $P_{UC}$ with the UC model, including all losses. These losses depend on the charge $Q(t)$ of the UC at that moment in the drive cycle. For all possible values of $P_{UC}$ the corresponding $P_{H2}$ is calculated with the FC model in the controller.
$P_{H2}$ and $P_{UC}$ times $s$ are added together and a minimum is determined, which leads to a value of $P_{UC}^o$ where the minimum fuel cost can be reached at a certain time on the drive cycle. This ECMS algorithm is visualised in figure 4.2.

Due to self-sustainability constraint of the UC the following equation needs to be satisfied at:

$$\int_{t=0}^{t_f} P_{UC}(t) \approx 0 \quad (4.11)$$

Figure 4.2: Principle of the ECMS algorithm.

4.4 Evaluation of the equivalent weight factor

The evaluation of the equivalence factor $s(t)$ is the core of the ECMS strategy. As becomes clear in Chapter 5, one should be able to adjust the equivalence factor if the UC is limited in its capacity. Because of its limited size the UC will be reaching its upper and lower boundaries on most of the drive cycles. That is the reason why different approaches to adapt the equivalence factor are investigated. Simulations are done on different drive cycles to see what is the best way for on-line implementation of the ECMS strategy in the “c,mm,n” vehicle.

4.4.1 Probability function

For a given drive cycle, different equivalence factors for charging and discharging can be calculated [9,15,20]. This is done by running simulations for various constant values of $P_{UC}$ over a whole drive cycle. After simulation the total fuel energy use is plotted against the electrical fuel use during charging and discharging. By fitting linear functions through the straight lines that appear, the different equivalence value $s_{chg}$ and $s_{dis}$ can be determined from the slopes of these lines.
The next step is to evaluate the equivalence factor $s(t)$ as a function of $s_{chg}$ and $s_{dis}$ with the probability function $p(t)$. This is done the following equations:

$$s(t) = s_{dis} p(t) + s_{chg} [1 - p(t)]. \quad (4.12)$$

$$p(t) = \frac{E_{UC}(t)}{t^b P_{FC_{max}}} + \frac{P_{EM}^{avg}}{P_{FC_{max}}} \quad (4.13)$$

The variable $E_{UC}(t)$ provides a feedback of the SOC of the UC, since it is the time integral of the UC power and thus describes the deviation in SOC from its initial value.

Time horizon $t^b$ has to be short to avoid deviations in the SOC beyond the limitations. This method has been implemented in the vehicle model, which did not lead to desired results, since the strategy proposed is extremely sensitive of variations in both equivalence factors and probability factor.

### 4.4.2 Penalty function

A penalty function is used to penalize the use of the UC when it has reach his lower limit and to penalize the use of the FC when the UC has reach his upper limit. There are different ways of integrating a penalty function in the control strategy presented in the literature. Always a heuristic control function is used on the SOC of the UC.

In [3] a function which penalizes the use of the FC is added to the cost function (4.7). A quadratic piecewise function is used to keep the UC charge away from its lower and upper boundary.

$$p(Q) = \begin{cases} \frac{Q(k) - (Q_{\text{min}} + \delta)}{Q_{\text{max}}}^2 & \text{if } Q(k) < (Q_{\text{min}} + \delta) \\ \frac{Q(k) - (Q_{\text{max}} - \delta)}{Q_{\text{max}}}^2 & \text{if } Q(k) > (Q_{\text{max}} - \delta) \\ 0 & \text{otherwise} \end{cases} \quad (4.14)$$

$\delta$ determines how far from the boundaries of the UC charge this penalty function becomes active.

Another non-linear function that penalizes the control power output of the UC uses the discrepancy between the reference relative value on the SOC of the UC $Q_{\text{ref}}$ and the actual relative value $Q(t)$.[11]

$$s(t) = \phi_1 \left( 1 - \left( \frac{Q_{\text{ref}} - Q(t)}{Q_{\text{max}} - Q_{\text{min}} / 2} \right)^{2\phi_3 + 1} \right) + \phi_2 \int_0^t (Q_{\text{ref}} - Q(\tau)) d\tau \quad (4.15)$$

This penalty function can be adapted via parameters $\phi_1, \phi_2$ and $\phi_3$. 
4.4.3 P(I) controller

Another method to use the discrepancy between a reference value and the actual value of $Q(t)$ of the UC is to calculate $s(t)$ online by using a P(I) controller.[13]

$$s(t) = s_0 + K_p (Q_{ref} - Q(t)) + K_i \int_0^t (Q_{ref} - Q(\tau)) d\tau$$

The I action in this controller with the tune parameter $K_i$ keeps track of the SOC of the UC, which mainly influences the amplitude of the low frequent oscillation of $Q(t)$. This low-frequent oscillations depend on the drive cycle the vehicle is simulated on. For the UC with the small content it may be enough to use a P controller because the charge $Q(t)$ changes faster from depleted to fully charged.

The solution is also very sensitive to the initial value $s_0$ and $K_p$. If $K_p$ is chosen too large, $s(t)$ fluctuates to much over the drive cycle, leading to decreased fuel economy of the vehicle. $s_0$ has influence on $s(t)$ on the whole cycle, determining the charge $Q(t)$ of the UC. It is therefore very important to have a good estimation if the initial value $s_0$ and a mildly tuned P controller which makes full use of the capacity of the UC. Figure 4.3 shows the feedback diagram for the P controller.

Figure 4.3: Feedback diagram for estimation of $s$ by using a P controller. [13]

4.4.4 Active reference value $Q_{ref}$

The reference value of the UC charge $Q_{ref}$ is a very important parameter. When it is chosen exactly between the maximum charge of 90% and minimum charge of 50%, one can do with a mildly tuned value of $K_p$ without reaching the boundaries if the UC. The minimum and maximum charge of the UC are reproduced from [21]. If $Q_{ref}$ can be changed during the cycle, for example depending on the speed of the vehicle, one should be able to influence the charge of the UC more accurate on the cycle. For example, a vehicle driving at 120 km/h does not need a fully charged UC while it contains a lot of kinetic energy to charge the UC when it decelerates. In the same way it is possible to charge the UC at slow speed to be able to accelerate with a charged UC for extra drive power. In Chapter 5 a proposal is done to determine this reference value during the drive cycle.
4.5 ECMS controller model

The ECMS controller in the vehicle model has three input parameters. The power required by the electric motor $P_{EM}(t)$, the charge in the Ultra Capacitor (UC) $Q_{UC}(t)$ and the voltage in the UC $U_{UC}(t)$.

![Figure 4.4: Controller model in QSS-Toolbox](image)

Figure 4.4 shows the ECMS model in the QSS-TB. Since the ECMS controller calculates the optimal power split at a certain moment, information on fuel consumption about both the Fuel Cell (FC) and the UC is needed at any instance. Therefore both models of these components need to be present in the controller model. The UC model is adapted so it is able to calculate with an input vector form like $P_{UC}$. This UC model is calculating its losses with use of the charge $Q(t)$ of the UC model in the drive train. The FC system is represented by a parabolic function, since the hydrogen power $P_{H2}$ is a parabolic function of the FC power $P_{FC}$. For any control value of $P_{UC}$, within the range of the UC, and charge $Q(t)$ the power of the UC is calculated $P_{UC}$. Multiplying these values, at any time, with the equivalence factor $s(t)$ results in a vector of values, which can be added to the hydrogen power $P_{H2}$. This power is a result of subtracting the control vector $P_{UC}$ from the required power of the motors $P_{EM}(t)$. The “FC limit” block takes care of the power range of the FC. In this way, only viable values of the optimum control vector $P_{UC}$ are calculated.

At this point the cost function is calculated using:

$$J(t) = P_{H2}(t) + s(t)P_{UC}(t)$$ \hfill (4.17)

The relational operator within the ECMS controller arranges a vector which consists of a one where the minimum value of $J(t)$ occurs. By taking the dot product of this vector with the vector $P_{UC}$, the value of $P_{UC}$ for which the minimum fuel consumption occurs, is passed. The block “Limit UC” looks after the allowable bound of the UC. Finally the optimal power split can be calculated using:

$$P_{EM}(t) = P_{FC}(t) + P_{UC}(t)$$ \hfill (4.18)
Figure 4.5 visualises the calculation of the minimum value of the cost function for two different situations. All the different controller rules are plotted as a function of the control vector $P_{UC}$. The upper subfigure shows a situation where the traction power of the in-wheel motors is a positive value of 14.8 [kW]. One can recognize the different terms in $J(t)$, from which the minimum is determined. It shows that the minimum fuel use, for the instant value of $s(t)$, is reached at a value of $P_{UC} = 9.2$ [kW]. From (4.5) this means that the FC has to deliver $14.8 - 9.2 = 5.6$ [kW] of power, without the auxiliaries taken into account. The lower subfigure shows a situation of regenerative braking. The drive power is -5.3 [kW], and a minimum fuel use occurs at $P_{UC} = -6.3$ kW. This means $-5.3 - (-6.3) = 1$ [kW] of power has to be delivered by the FC.

Figure 4.5: ECMS controller minimum determination
Chapter 5: Simulation results

The previous chapter showed multiple solutions for Energy Management (EM) strategies. This chapter presents the simulation results of EM strategies from both the off-line and on-line type. In section 5.1 the results of the ECMS with constant equivalence factor are presented. Section 5.2 presents the results of the adaptation of the equivalence factor with the P controller. Component sizes are varied by taking into account vehicle weight and fuel consumption in section 5.2.1. Also the sensitivity of the variation in control parameters is explored. The ECMS with P controller is simulated on different drive cycles and under different circumstances in section 5.2.2 and 5.2.3. The intention of section 5.3 is to provide insight in the performance of the real-time EM strategy in comparison with an optimal solution when it comes to fuel economy. Hence, a proposal is done in section 5.4 to increase the performance of the ECMS by tuning the reference value $Q_{ref}$ on the cycle.

5.1 ECMS results with constant equivalence factor

In section 4.3.1 the ECMS was explained. The first part of the investigation focuses on finding a solution for a constant equivalence factor $s$ on a specific drive cycle. For the drive cycle, the NEDC is chosen, since it is used in most of the literature on EM strategies. All new vehicles are also tested on fuel economy on the NEDC. Figure 5.1 shows the speed profile of the NEDC. The lower subfigure presents the power that has to be delivered by the in-wheel motors to follow the speed profile of the drive cycle. The vehicle main parameters are listed in table 5.1. The road load equation are described in Section 3.1.

<table>
<thead>
<tr>
<th>Vehicle mass $m_v$</th>
<th>900 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal area $A_f$</td>
<td>2.1 $m^2$</td>
</tr>
<tr>
<td>Roll drag $c_r$</td>
<td>0.01</td>
</tr>
<tr>
<td>Air drag $c_d$</td>
<td>0.23</td>
</tr>
<tr>
<td>Wheel radius $R_w$</td>
<td>0.6 $m$</td>
</tr>
</tbody>
</table>

Table 5.1 Vehicle parameters.
To find a solution for the equivalence factor, the vehicle is first simulated with the most fuel efficient component sizes which were calculated with DP. For the Fuel Cell (FC) this means a 50 [kW] stack and a 30 [kW] Ultra Capacitor (UC) module consisting of 92 units with a total capacity of 54.34 [F]. The result can be seen in figure 5.2. The parameters are plotted according figure 3.1.

The EM strategy takes care of a good power split between the FC and the UC. However figure 5.2 clearly shows that the State-Of-Charge of the UC reaches its admissible bounds. This phenomenon is due to the low capacity of the UC which is quickly fully charged or fully discharged.
discharged. As a consequence, energy is dissipated by the brakes instead of being recovered. It is also responsible for the jerky behaviour of the FC and UC. The power delivered by the FC is cut off when the UC is fully charged. At this point all the power demanded, is delivered by the UC. The FC at that moment is instantaneously running at idle. The next step is to determine the constant equivalence factor \( s \), where the end constraint is satisfied:

\[
\Delta SOC|_{t_f} = \frac{Q(t_f) - Q(0)}{Q(0)} = 0
\]  

(5.1)

Therefore the final SOC is expressed as a function of \( s_0 \). A wide range of values of \( s_0 \) is chosen to see what the constant value of \( s_0 \) is, for which (5.1) holds. The initial SOC is set at 70\% (between the maximum charge of 90\% and minimum charge of 50\%) to be able to react on any opportunity of assisting or regenerating. These values are taken over from [21], to get comparable results. Thereafter the range is limited around values of \( s_0 \) where the \( \Delta SOC \) approaches zero. Figure 5.3 shows the results of the function with steps of 3e-4 in \( s_0 \). From this figure one can see there exist multiple solutions for \( s \), since the last deceleration in the NEDC charges the UC to a certain level. Because the UC is depleted for many values of \( s_0 \) when this charging begins, the end state of the UC will only vary because of the value \( s_0 \). The noise occurs while different values of \( s_0 \) determine various power split optimums over the whole cycle and therefore also the exact moment of charging the UC at the end of the cycle. So one can find one unique solution for the equivalence factor \( s \) with the ECMS strategy only when the operating boundaries of the UC are never crossed.

![Figure 5.3: \( \Delta SOC \) as a function of \( s_0 \) for a 54.34 [F] UC.](image)

To confirm this hypothesis, a simulation is done with the same vehicle but now equipped with a larger UC. A simulation shows that three parallel UC modules are able to run the NEDC without reaching the SOC limits. Now the UC has a capacity of 163.02 [F]. The results are presented in figure 5.4. The ECMS takes care of all the drive power to be delivered by the UC. A small amount of power delivered by the FC, depending on the equivalence factor \( s_{0f} \), is used to charge the UC.
To be able to find a solution without the UC reaching its bounds, the initial charge in the UC is lowered from midpoint. Again $\Delta SOC$ is plotted versus $s_0$, with the same accuracy of $3 \times 10^{-4}$ in $s_0$. Figure 5.5 exhibits a nearly smooth behaviour and $s_0$ can be determined easily. For $s_0 = 1.2356$, (5.1) is satisfied. At $s_0$ around 1.22 one can see the same behaviour as with the small UC. This is due to the lower value of $s_0$, which is responsible for depleting the UC on the last acceleration of the NEDC. The results show that a solution for the equivalence factor asks for a change in component size for the UC. On the one hand it is beneficial to use a large UC, which does not reach its allowable bounds, on the other hand a large UC leads to extra weight and volume. By adding extra capacity to the UC it although becomes less demanding for the FC, which can be downsized.

This is further explained in Section 5.2.1, where the FC size is varied together with the UC.

**Figure 5.4:** 50 [kW] FC, 163 [F] UC, ECMS with constant $s$.

**Figure 5.5:** $\Delta SOC$ as a function of $s(0)$ for a 163.02 [F] UC.
5.2 ECMS results with P controller on equivalence factor

Since the Ultra Capacitor (UC) in the “c,m,m,n” is limited in its capacity and it is not preferable for an UC to reach its upper or lower bound, a P controller on the equivalence factor $s$ is added to the Equivalent Consumption Minimization Strategy (ECMS). This controller takes care of the charge $Q(t)$ in the UC, by keeping it near a reference value $Q_{ref}$ (Fig. 4.3). Within this strategy three tuning parameters are now present: $s_0$, the initial equivalence factor, $K_p$, the gain of the proportional controller, $Q_{ref}$, the reference value for the UC charge.

$s_0$ is determined in the same way as in the previous section. With the P controller added to the ECMS strategy, the UC does not cross its bounds so a solutions for $s$ can be determined. $Q_{ref}$ is chosen at 70%, where the charge of the UC is between the maximum charge of 90% and minimum charge of 50%. The initial value $Q(0)$ is set near the end value $Q(t_f)$ so $\Delta SOC = 0$. By doing this, it is not necessary to compensate electrical energy to chemical energy when $\Delta SOC \neq 0$, and fuel economy can easily be compared with other vehicles on the same drive cycle. This initial value $Q(0)$ does not influence the behaviour of the controller during the cycle. Finally, $K_p$ must be chosen very mild to make full use of the UC. However, if this value is too mild, the UC will reach its allowable bounds. First the optimal component sizes of 50 [kW] for the FC and 30 [kW], 54.34 [F] for the UC is simulated. The results are shown in figure 5.6.

![Figure 5.6: ECMS results with 50 [kW] FC, 54.34 [F] UC.](image)

**Controller parameters:** $K_p=0.009$, $Q_{ref}=70\%$ and $s_0=1.862$.

The results exhibit the desired behaviour. The UC is not fully charged during the cycle and it allows recovering all the kinetic energy. This should result in increased fuel economy. Also note the smooth behaviour of the FC. The UC takes care of all the peak power demands, while the FC is running at low power with no strong fluctuations. This is exactly what the EM strategy should do.
Figure 5.7 shows the equivalence factor $s(t)$ over time. It is clear, when comparing with figure 5.6, that the profile that occurs is related to the charge $Q(t)$ in the UC. Since the difference between the actual charge $Q(t)$ and the reference value $Q_{\text{ref}}$ determines how much the equivalence factor $s$ is adapted. The control value $K_p$ stays constant over the whole cycle and is only responsible for the range of $s$, figure 4.3. In the first part of the cycle, the equivalence factor is around 1.5, which is the value were the charge stays just under 90%. This is necessary for the last “hill” in the NEDC, which depletes the UC when it is not fully charged. This last “hill” also causes the increase of $s(t)$ at the end of the cycle. Since, more power has to be delivered by the FC, the power split is shifted towards the FC with this increased value. The value of $K_p$ influences the spread of $s(t)$ over the cycle, where a mildly tuned controller keeps the equivalence factor near its initial value $s_0$ for better fuel efficiency. This is explained in the next session.
5.2.1 Optimization of component sizes.

To determine the sensitivity of the control parameters for different component sizes, simulations with different power train configurations are done. One can also check whether the component sizes of 50 kW Fuel Cell (FC) and 30 [kW], 54.34 [F] Ultra Capacitors (UC) are the most fuel efficient. The component weight variation is taken into account. The weight increase is calculated according table 3.1, 3.2 and 3.3. The FC is varied between 20 [kW] and 60 [kW], while extra UC modules are set parallel to increase its capacity. The component sizes with the best fuel economy, are simulated more accurate as one can see in the black border of table 5.2. All the simulation results of these configurations can be seen in the appendix. The configuration with a 20 [kW] FC and 1 UC could not provide enough power to complete the NEDC.

When simulating these different component sizes, the tuning parameters have to be changed. $K_p$, for example, has to be decreased when the UC capacity increases. Otherwise the EM strategy does not make full use of the UC capacity. The different settings are presented in table 5.3. For each UC module a value of $K_p$ is chosen which allows the strategy to make full use of its capacity. The initial value $Q(0)$ has to be varied to make sure $\Delta SOC = 0$, which is necessary for comparing the fuel use. The values of $s_0$ for each combinations are the initial equivalence factors for the UC capacity does not cross its bounds.

### Table 5.2: Simulation results for fuel consumption [MJ]

<table>
<thead>
<tr>
<th>$N_{UC,par}$</th>
<th>FC 20 kW</th>
<th>30 kW</th>
<th>40 kW</th>
<th>45 kW</th>
<th>50 kW</th>
<th>60 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td></td>
<td>9.190</td>
<td>9.181</td>
<td>9.233</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td>9.115</td>
<td>9.089</td>
<td>9.071</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Chapter 2 is decided that the combined electrical power of the FC and UC should at least be 60 [kW]. It is also considered that the FC and UC both should have a size of 30 [kW]. From figure 4.3 can be seen that a FC has a higher efficiency at partial load, which results in a reduction of fuel consumption. On the other hand, increasing the FC size will increase the vehicle weight, which causes higher fuel consumption. One can see from table 5.2 that an increase of FC size will lead to a decrease in fuel consumption up to a certain point where the increase in vehicle weight stops the decrease of fuel consumption. An increase of UC size causes the FC to work in a more efficient area, leading to a better fuel efficiency. Figure 5.8 shows the fuel consumption and weight for each drive train configuration. The best fuel economy is reached with a 50 [kW] FC and 2.5 parallel UC module, in spite of the vehicle weight. This is also the configuration where no P controller is needed to avoid the UC to reach its charge limits. If one has to choose for one UC module, the best fuel economy is reached with the 40 [kW] FC.
Figure 5.8: Fuel economy and weight for all power train configurations

The difference in fuel consumption between a 2.5 and 3 parallel UC is caused by the weight increase due to the larger UC. The increased capacity does not influence the FC size anymore because a 2.5 parallel UC is able to run the NEDC already without reaching its boundaries. From figure 5.8 can be seen that the difference in fuel consumption for various power train configurations is less than 3.5%.

<table>
<thead>
<tr>
<th>UC</th>
<th>FC</th>
<th>40 kW</th>
<th>45 kW</th>
<th>50 kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{UC,par} =1 )  ( K_p =0.009 )  ( Q(0) =80% )</td>
<td>( s_0 =1.851 )</td>
<td>( s_0 =1.856 )</td>
<td>( s_0 =1.862 )</td>
<td></td>
</tr>
<tr>
<td>( N_{UC,par} =1.5 )  ( K_p =0.0075 )  ( Q(0) =73% )</td>
<td>( s_0 =1.782 )</td>
<td>( s_0 =1.786 )</td>
<td>( s_0 =1.790 )</td>
<td></td>
</tr>
<tr>
<td>( N_{UC,par} =2 )  ( K_p =0.005 )  ( Q(0) =67% )</td>
<td>( s_0 =1.706 )</td>
<td>( s_0 =1.707 )</td>
<td>( s_0 =1.710 )</td>
<td></td>
</tr>
<tr>
<td>( N_{UC,par} =2.5 )  ( K_p =0 )  ( Q(0) =60% )</td>
<td>( s_0 =1.581 )</td>
<td>( s_0 =1.582 )</td>
<td>( s_0 =1.583 )</td>
<td></td>
</tr>
<tr>
<td>( N_{UC,par} =3 )  ( K_p =0 )  ( Q(0) =60% )</td>
<td>( s_0 =1.582 )</td>
<td>( s_0 =1.582 )</td>
<td>( s_0 =1.584 )</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.3: Parameter settings for simulations
Table 5.3 shows that the initial equivalence factor $s_0$ decreases when the UC capacity increases. A decrease of this initial value $s_0$ means that more power is delivered by the UC over the whole cycle because it is “cheaper”. This makes sense while a larger UC contains more energy. A small increment in $s_0$ occurs when the FC is enlarged. This means less power has to be delivered by the same UC in combination with a larger FC. The power split is shifted towards the FC because the larger FC is working in a more efficient area.

Looking at the maximum power that has to be delivered by the FC, one can see a major decrease in power for a larger capacity of the UC. Figure 5.9 upper subfigure. This means that a power train with larger UC is able to run the NEDC with a smaller FC. If this is translated to the cost price of the whole power train, with the estimated cost price from Chapter 3, a trade off become visible. A 23 [kW] FC with 1 UC and an 8 [kW] FC with a 2.5 parallel UC both decrease the cost price, while the small FC with the largest UC are the most cost efficient. Figure 5.9 lower subfigure.

![Figure 5.9: Maximum fuel cell power and power train cost price](image)

The fuel economy shows a decrease for a smaller FC size, table 5.4. This is due to the low efficiency of the FC when its output power is relatively high. In this simulation the FC was just the size of the maximum power demand on the cycle.

The estimated cost price has to be taken into account, while this is a global estimation based on actual numbers from the internet and manufacturers [1,16,18]. Through mass production and less expensive raw materials this cost price might differ from the estimation.

<table>
<thead>
<tr>
<th>Power train configuration</th>
<th>Fuel consumptions [MJ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.5 kW FC + N_{UC,par} =1</td>
<td>9.704</td>
</tr>
<tr>
<td>19.5 kW FC + N_{UC,par} =1.5</td>
<td>9.783</td>
</tr>
<tr>
<td>15.0 kW FC + N_{UC,par} =2</td>
<td>9.956</td>
</tr>
<tr>
<td>8.0 kW FC + N_{UC,par} =2.5</td>
<td>11.01</td>
</tr>
</tbody>
</table>

**Table 5.4: Fuel consumption of downsized fuel cell power train.**
5.2.2 ECMS under various conditions

To examine the influence of weight increase on the different drive train configurations with their control parameters, the following table of simulation possibilities is completed. The most fuel efficient configurations from 5.2.1 are submitted to a significant weight increase. One adult is estimated to weigh 80 [kg], a child 40 [kg] and luggage 20 [kg] per person.

<table>
<thead>
<tr>
<th>Additional weight</th>
<th>40 kW FC</th>
<th>45 kW FC</th>
<th>50 kW FC</th>
<th>50 kW FC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N\textsubscript{UC,par}=1 \ K\textsubscript{p}=0.009</td>
<td>N\textsubscript{UC,par}=1.5 \ K\textsubscript{p}=0.0075</td>
<td>N\textsubscript{UC,par}=2 \ K\textsubscript{p}=0.005</td>
<td>N\textsubscript{UC,par}=2.5 \ K\textsubscript{p}=0</td>
</tr>
<tr>
<td>80 kg: 1 Adult</td>
<td>940 kg s\textsubscript{0}=1.873 Fuel: 9.459 MJ</td>
<td>975 kg s\textsubscript{0}=1.804 Fuel: 9.350 MJ</td>
<td>1010 kg s\textsubscript{0}=1.726 Fuel: 9.377 MJ</td>
<td>1025 kg s\textsubscript{0}=1.587 Fuel: 9.232 MJ</td>
</tr>
<tr>
<td>160 kg: 2 Adults</td>
<td>1020 kg s\textsubscript{0}=1.887 \ \textbf{K\textsubscript{p}=0.0095} Fuel: 9.624 MJ</td>
<td>1055 kg s\textsubscript{0}=1.821 \ \textbf{K\textsubscript{p}=0.008} Fuel: 9.534 MJ</td>
<td>1090 kg s\textsubscript{0}=1.750 \ \textbf{K\textsubscript{p}=0.006} Fuel: 9.437 MJ</td>
<td>1105 kg s\textsubscript{0}=1.656 \ \textbf{K\textsubscript{p}=0.003}</td>
</tr>
<tr>
<td>200 kg: 2 Adults + 1 Child</td>
<td>1060 kg s\textsubscript{0}=1.889 \ \textbf{K\textsubscript{p}=0.010} Fuel: 9.739 MJ</td>
<td>1095 kg s\textsubscript{0}=1.835 \ \textbf{K\textsubscript{p}=0.008} Fuel: 9.616 MJ</td>
<td>1130 kg s\textsubscript{0}=1.762 \ \textbf{K\textsubscript{p}=0.0065} Fuel: 9.580 MJ</td>
<td>1145 kg s\textsubscript{0}=1.671 \ \textbf{K\textsubscript{p}=0.0035}</td>
</tr>
<tr>
<td>240 kg: 2 Adults + 2 Children</td>
<td>1100 kg s\textsubscript{0}=1.901 \ \textbf{K\textsubscript{p}=0.010} Fuel: 9.815 MJ</td>
<td>1135 kg s\textsubscript{0}=1.838 \ \textbf{K\textsubscript{p}=0.0085} Fuel: 9.718 MJ</td>
<td>1170 kg s\textsubscript{0}=1.770 \ \textbf{K\textsubscript{p}=0.0065} Fuel: 9.628 MJ</td>
<td>1185 kg s\textsubscript{0}=1.686 \ \textbf{K\textsubscript{p}=0.004}</td>
</tr>
<tr>
<td>320 kg: 4 Adults</td>
<td>1180 kg s\textsubscript{0}=1.914 \ \textbf{K\textsubscript{p}=0.0105} Fuel: 10.01 MJ</td>
<td>1215 kg s\textsubscript{0}=1.858 \ \textbf{K\textsubscript{p}=0.0085} Fuel: 9.904 MJ</td>
<td>1250 kg s\textsubscript{0}=1.789 \ \textbf{K\textsubscript{p}=0.007} Fuel: 9.812 MJ</td>
<td>1265 kg s\textsubscript{0}=1.706 \ \textbf{K\textsubscript{p}=0.0045}</td>
</tr>
<tr>
<td>400 kg: 4 Adults + Luggage</td>
<td>1260 kg s\textsubscript{0}=1.927 \ \textbf{K\textsubscript{p}=0.011} Fuel: 10.21 MJ</td>
<td>1295 kg s\textsubscript{0}=1.875 \ \textbf{K\textsubscript{p}=0.009} Fuel: 10.08 MJ</td>
<td>1330 kg s\textsubscript{0}=1.807 \ \textbf{K\textsubscript{p}=0.0075} Fuel: 9.995 MJ</td>
<td>1345 kg s\textsubscript{0}=1.727 \ \textbf{K\textsubscript{p}=0.005}</td>
</tr>
</tbody>
</table>

Table 5.5: Simulation results for increased vehicle load

The power increase due to higher loads lead to higher values of the equivalence factor \(s(t)\). So when more power is needed to drive the vehicle over a certain drive cycle, the power split is shifted toward the UC to get the same balance as for lower loads. This, as expected, holds for each configuration. Due to the higher power demand from the UC, the \(K\text{p}\) value of the controller needs to be adjust to avoid the UC to reach its bounds. The values of \(K\text{p}\) and \(s\text{0}\) as a function of the increased vehicle weight are presented in figure 5.10. One can notice the need to adjust the parameter settings in the EM controller to make full use of the UC when the vehicles weight increases.
Obviously there is a relation between the value of $s_0$ an $K_p$. Equal profiles occur when plotting them against the increased vehicle weight and thereby the increased drive power. When the values of $s_0$ and $K_p$ are plotted against each other, figure 5.11, one can see the relation between the two control parameters. There exists almost a linear function, which is obvious when one realises both parameters influence each other within the controller. A higher value of $s_0$ means less power is delivered by the UC, due to the small capacity which is also responsible for the higher control value $K_p$, to keep the charge near the reference value $Q_{\text{ref}}$.

For implementation in a real vehicle, these values have to be taken into account, while keeping in mind that these are only valid on the NEDC. Though, it is expected that these adaptations to the control parameters are necessary when the power demand from the electric motors increase. To confirm this hypothesis, the EM strategy is subjected to a drive cycle which demands more drive power in the next section.

5.11: $s_0$ vs $K_p$. 
5.2.3 ECMS on different drive cycle

To check whether the ECMS strategy with the P controller complies with driving conditions which demand more power, the most fuel efficient configurations from 5.2.1 are simulated on the SFTP-US06 cycle. This cycle is a real world highway cycle which demands a higher power output from the power train, figure 5.12. The results from the simulations are presented in table 5.6.

<table>
<thead>
<tr>
<th>UC</th>
<th>FC [kW]</th>
<th>$s_0$</th>
<th>Fuel consumption [MJ]</th>
</tr>
</thead>
</table>
| $N_{UC,par} = 1$  
$K_p=0.011$  
$Q(0)=89\%$  | 40      | 2.022  | 13.75                  |
| $N_{UC,par} = 1.5$  
$K_p=0.0075$  
$Q(0)=88\%$  | 45      | 1.992  | 13.74                  |
| $N_{UC,par} = 2$  
$K_p=0.005$  
$Q(0)=84\%$  | 50      | 1.999  | 13.29                  |
| $N_{UC,par} = 2.5$  
$K_p=0$  
$Q(0)=78\%$  | 50      | 1.976  | 13.29                  |

Table 5.6: Simulation results for SFTP-US06

The results make clear that the fuel economy increases when the power train component sizes are increased. The same results as for the NEDC. This is due to the higher efficiency of the Fuel Cell (FC) when working on partial load. The value of $s_0$ decreases for larger drive train components, which is necessary to balance the power split between the FC and the Ultra.
Capacitor (UC). In this case, more power is used from the UC because of its enlarged capacity.

What also strikes is that only for the smallest configuration, consisting of a 40 [kW] FC and one UC, the value of \( K_p \) needs to be adjust in the controller. It is therefore not possible to drive this configuration on both the NEDC and US06 cycle without tuning \( K_p \).

In the previous section, the same need for adaptation in the controller for higher loads was present, but in that case for all configurations.

The value of \( s(0) \), for the larger components, is the only parameter that needs to be increased for the different controllers to handle the higher power demand of the US06 drive cycle. \( K_p \), which is matched to the number of parallel ultra capacitors can be unchanged. Figure 5.13 presents the simulation results for the smallest power train configuration.

![Figure 5.13: US06 results with 40 kW fuel cell, 54.34 F ultra capacitor](image)

Controller parameters: \( K_p=0.011 \), \( Q_{\text{ref}}=89\% \) and \( s(0)=2.022 \).

The real world origin of this drive cycle makes the delivered power by the FC and UC more jerky. But the overall objective of the EM strategy remains, since the UC takes care of the peak power demands from the drive cycle.
5.3 ECMS vs. DP

In order to investigate the performance of the proposed Energy Management (EM) strategy a comparison is made. The simulations cover four different strategies from Chapter 4. The first simulation refers to a baseline strategy to see the decrease in fuel consumption after hybridization. A model is simulated with no Ultra Capacitor (UC), using only the Fuel Cell (FC) to power the vehicle. No brake energy recuperation and no power loss from the FC to charge the UC. Figure 5.14 shows the hydrogen consumption of the FC stack and the power demand at the FC. One can see the lower efficiency of the FC at higher loads. (Chapter 3). This results in an energy consumption of 10.76 [MJ] for the NEDC with a 60 [kW] FC stack.

Figure 5.14: 60 [kW] FC on NEDC

The second simulation refers to the Equivalent Consumption Minimization Strategy (ECMS) with the constant equivalence factor, where the small UC crosses its admissible bounds. Simulation 3 covers the same vehicle with a P controller added to the ECMS to control the UC charge. The last simulation presents the results from [21], where the optimal solution is calculated with a Dynamic Programming (DP) routine. The overall fuel consumption for each strategy is listed in table 5.7.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Fuel use [MJ] (absolute)</th>
<th>Fuel use [%] (relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>10.76</td>
<td>100</td>
</tr>
<tr>
<td>ECMS (s constant)</td>
<td>10.44</td>
<td>97.0</td>
</tr>
<tr>
<td>ECMS with P controller</td>
<td>9.229</td>
<td>85.8</td>
</tr>
<tr>
<td>Dynamic Programming</td>
<td>8.608</td>
<td>80.0</td>
</tr>
</tbody>
</table>

Table 5.7: Fuel consumption over NEDC

It shows that DP achieves the absolute lowest fuel consumption, as expected. What strikes most is that ECMS with the constant equivalence factor is not much more beneficial over the cycle than the baseline vehicle. This can be blamed to the increased vehicle weight due to hybridization. The reduction, compared to the baseline vehicle, for DP is 20 % against a reduction 14.2 % for the ECMS with P controller.
In figure 5.15 the State-Of-Charge (SOC) in the UC is presented for the different strategies. From the SOC trajectory it is clear that the DP strategy shows the most beneficial behaviour. The UC is fully charged at the moment when it has to deliver most power. This is due to the fact that the optimal trajectory can be calculated with knowledge of the future charge and discharge opportunities. This is different for the ECMS where the optimal power split is calculated at each time step. The P controller takes care of the admissible bounds of the UC, while the equivalence factor \( s(t) \) ensures the recovery of energy in the UC. \( s(t) \) is also responsible for the higher charge level in the UC at the end of the cycle for the ECMS with P controller. The behaviour of the ECMS with constant equivalence factor shows an inefficient use of the UC. For a constant value of \( s \), the upper and lower bound are reached.

![Simulation results for S2, S3 and S4.](image1)

Figure 5.15: Simulation results for S2, S3 and S4.

If the constant equivalence factor within the ECMS is calculated with a larger UC (163 F) one can see similar behaviour of the SOC trajectory. Figure 5.16 shows the absolute charge in the UC for both ECMS and DP. It is clear that when the admissible bounds of the UC are not crossed with the on-line ECMS, it is tracking the optimal solution calculated with off-line DP strategy. Looking at the fuel consumption for both strategies, a difference occurs. 8.608 [MJ] for DP and 9.071 [MJ] for ECMS. This difference is due to the higher weight level of the power train for ECMS. The power train needs a 2.5 parallel UC to be able to run the NEDC without reaching the boundaries of the UC. Also the different power split decisions between both strategies contribute to the different energy consumption.

![ECMS with optimal s(0) compared with DP](image2)

Figure 5.16: ECMS with optimal s(0) compared with DP

Looking at the power split between the FC and UC for the ECMS with P-control and DP, figure 5.17, the different power split at the higher power demands strikes. More power is delivered by the FC for ECMS and more by the UC by DP. This is due the on-line calculation of ECMS. It cannot foresee the higher power demand at the end of the cycle, where DP can always use the FC in the most efficient (low power) region.
The vertical row of power split opportunities which occur at 7.5, 15 and 23 [kW], are related to the P controller in ECMS. The value of $s(t)$, which manages the power split, react on the P controller. So the different SOC levels for the same power demand causes multiple solutions for $s(t)$. In the figure it is also visible that for a number of occasions, the FC is producing more power than necessary in order to charge the UC. This behaviour is almost similar for ECMS and DP.

![Figure 5.17: Power split between FC and UC for the ECMS with P controller (top) and DP](image)
5.4 Results with active reference value $Q_{\text{ref}}$

At the end of chapter 3, a proposal is done for making the reference value $Q_{\text{ref}}$, dependent on the vehicle speed. In this way it should be possible to influence the value for the charge $Q(t)$ in the UC during the cycle. Figure 5.18 shows a profile which is simulated on the NEDC. For high speed driving the reference is set to zero, since a lot of kinetic energy is available in the vehicle, which gives the opportunity of charging the UC at deceleration.
At low speeds the reference is set to “full charged” while acceleration of the vehicle demands more power from the drive train. Therefore it is more beneficial to have a charged UC in this situation. The parabolic function is determined by “trial and error” to get the desired behaviour of the relation between the vehicle speed $v(t)$ and the reference value $Q_{\text{ref}}(t)$.

![Figure 5.18: $Q_{\text{ref}}$ as function of the vehicle speed](image)

The results obtained with the additional active value for $Q_{\text{ref}}$, are not satisfactory. The value of $K_p$ had to be increased due to the smaller band that occurs between the reference value $Q_{\text{ref}}$ and the upper and lower bound of the UC. At zero speed, the reference value is set to high, which resulted in an enormous immediate charge by the FC. Also the charge level at 80 [km/h] is set too low, causing the UC to be depleted before the last “hill” of the NEDC is finished. To overcome this problem the profile of figure 5.19 is introduced in the simulation model.

![Figure 5.19: $Q_{\text{ref}}$ as function of the vehicle speed](image)
Here, the value at zero speed is lowered and the margins between $Q_{ref}$ and the upper and lower bound are expanded. Also the value for 80 [km/h] is increased to be able to run the whole cycle without depleting the UC. Figure 5.20 shows the simulation results for the charge in the UC. One can see that the admissible bounds are not crossed and that the profile that occurs is more similar to the DP solution. Since the value of $K_p$ is increased from 0.009 to 0.013, the equivalence factor $s$ fluctuates more. In this way, the optimum value is more or less shifted away in order to stay at the desired charge level $Q(t)$. This can cause the small increase in fuel consumption, from 9.229 [MJ] to 9.269 [MJ]. A proposal for a better solution is done in Chapter 6.

![Figure 5.20: Charge $Q(t)$ with the additional profile of 5.19. A 40 [kW] FC and 1 UC configuration is used. Parameter $K_p$ is increased from 0.009 to 0.013.](image)
Chapter 6: Conclusions and recommendations

This final chapter provides an overview of the conclusions that originate from the research described in this report. The first section discusses the conclusions of this research, where the second section will provide the recommendations for further research.

6.1 Conclusions

In literature, many solutions have been proposed for real-time Energy Management (EM) strategy for Fuel Cell Hybrid Vehicles (FCHV). The Equivalent Consumption Minimization Strategy (ECMS) was chosen for this assignment to investigate the on-line strategy performance in the “c.m.m.n”. This choice is based on a literature study proposed in Chapter 3, where the most straight forward and best implement able on-line EM strategy was elected. The QSS-Toolbox has been used for simulation purpose and the different component models were discussed in Chapter 4. The QSS-Toolbox is a good tool for simulation purpose, while it is possible for power train systems to be designed quickly and in a flexible manner, to easily calculate the fuel consumption of such systems.

In Section 5.1, the first simulation results of the ECMS are proposed. It shows, that the optimized component size from Dynamic Programming (DP) for the Ultra Capacitor (UC) is not capable of running the NEDC without reaching its bounds. A constant equivalence factor $s$ can not be determined when the admissible bounds of the UC are reached. Multiple solutions that exist in this situation. The solution to this problem is either to increase the capacity of the UC or using a P controller on the equivalence factor that uses the State-Of Charge (SOC) of the UC as its reference.

Section 5.2 shows the results of the ECMS with P controller. The enlargement of the UC does not appear to be a good solution, since it brings along extra weight and volume. By adding a P controller on the equivalence factor, the UC charge can be controlled within its margins when the right control parameters are used. Where the equivalence factor, for a large UC can be limited to one constant value $s$, the equivalence factor with the P-controller varies over the whole drive cycle ($s(t)$). Therefore the solution, for which the cost function is minimized, is being affected by the P-controller.

This imposed minimum shows a decrease in fuel efficiency in sections 5.2.1 and therefore another solution for the optimal component sizes than calculated with (DP). Other findings are the different values for controller gain $K_p$ and the initial equivalence factor $s_0$ for the different power train configuration. These adaptations are necessary to make full use of the UC. A larger UC is responsible for the increased margins on depleting or over-charging, which leads to lower values of $K_p$. The increased weight and thereby the increased power demand causes the value of $s_0$ to decrease. More power is being delivered by the UC, which is possible due to the high amount of energy available. Fuel consumption is related to a combination of components in the power train configuration. A large Fuel Cell (FC) is working in a more efficient area and since extra weight, of the increased FC, leads to more fuel consumption an optimum can be reached. The UC contributes to the fuel efficiency while it keep the FC on partial loads. Although the extra weight from the enlargement has to be taken into account as well. A larger UC leads to a lower maximum power demand for the FC, which gives the opportunity to downsize for cost efficiency. The estimated cost prices for the
component show a benefit for either a large UC with a small FC or a small UC with a large
FC. The choice to divide the component sizes is turning out to be less efficient when it comes
to cost prices. Downsizing the FC is at the expense of the fuel efficiency due to the higher
maximum power demand.

In section 5.2.2 en 5.2.3, the ECMS is subjected to higher power demands due to increased
vehicle weight and driving on a highway cycle. The results from the simulations show that the
ECMS can not handle these higher power demand without tuning its parameter settings. The
values of $K_p$ for the most fuel efficient drive train configuration need to be raised to keep the
charge of the UC within its acceptable bounds since more energy is used. The value of $s_0$ has
to be increased as well, so the charge level within the UC is raised. Due to higher power
demands, the power split is shifted towards the FC to contribute to the energy savings in the
UC. Plotting these two controller parameters, the relation between them becomes clear. There
exist almost a linear function between the enlargement of these two parameters, which is
obvious when one realises both parameters influence each other within the controller. A
higher value of $s_0$, means less power is delivered by the UC, due to its limited capacity. This
is also responsible for the higher control value $K_p$, in order to keep the charge near the
reference value $Q_{ref}$.

Section 5.3 is devoted to the comparison with a non-hybridised FC vehicle and with the
optimal EM solution calculated with DP. As stated before, the optimal component sizes on
weight and fuel economy differ from DP by using an adaptation on the equivalence factor $s(t)$.
DP is calculating with knowledge about the different opportunities for (dis)charging the UC
on the drive cycle. Therefore, the ECMS is not capable of following the optimal trajectory for
the UC charge $Q(t)$. The non-hybridised FC vehicle is used as a baseline to explore the
performance on fuel economy for both strategies. It is no surprise that the optimal DP
algorithm shows the best fuel efficiency by using 80% of the baseline fuel consumption.
ECMS uses 85% of the baseline fuel consumption. The ECMS can mimic the charge
trajectory result from DP. An increase in UC size is necessary to make use of a constant
equivalence factor $s$ and be able to run on the NEDC without reaching the boundaries of the
UC. This drive train configuration cannot compete with the DP algorithm on fuel economy
due to the increased weight of the UC module.

In an attempt to increase the performance of the ECMS strategy, a proposal for a more
accurate reference value for the P controller is explored in section 5.4. By making the value of
$Q_{ref}$ dependent on the vehicle speed, one is able to adjust the charge $Q(t)$ to the kinetic energy
available in the car. Since the simulations show a profile of $Q_{ref}$, that is very dependent on the
drive cycle, this solution is very sensible for the driving conditions of the vehicle. The value
of $K_p$ has to increase due to the decreased margins between the reference value and the upper
and lower limit of the UC. Therefore this solution can be different from the optimal power
split, leading to higher fuel consumption.
6.2 Recommendations

In the first place, the theoretical results obtained within this research are useful, but they have to be verified with hardware. The simulations performed within this assignment are a good tool to explore the possibilities of different EM strategies, but difficult to compare with driving a car in a real world environment. The NEDC is used for testing the fuel economy of new passenger cars in Europe, while the SFTP-US06 cycle shows a real world driving cycle. Within the simulations it is necessary to track the imposed drive cycles at any time. In real-world though, it appears often that the performance of a vehicle does not demand the drivers desire. In that case the acceleration is slower or top speed is decreased. The drive cycles within the simulations are imposed and need to be traced to get the right results. Rejection takes place if the power train is not able to meet the demands. A rejected drive train configuration can be accepted in real world under this sentence.

To be able to implement the ECMS in a prototype car, the control parameters need to be adjust under various circumstances as stated in Chapter 5. When the drive power is increased due to an enlargement of vehicle weight or higher power demanding driving conditions, the value of \( K_p \) and \( s_0 \) have to be tuned. If it is possible to make these parameters dependent on the average drive power at the electric motors, one should be able to keep the charge of the UC between its bounds, with a certain clearance, under any driving condition.

Another recommendation can be done on the active reference value \( Q_{ref} \). In this assignment a proposal is done, by making this value dependent on the vehicle speed. It seems this can be done in a more proper way by looking at acceleration and cruising of the vehicle. By making use of a rule-based control algorithm it is possible to distinguish whether the vehicle is accelerating or driving at a constant speed. While accelerating a vehicle demands the highest power train output power it is preferable to have a fully charged UC under this condition. For a cruising vehicle the power demand is significantly lower. By making a distinction between these driving modes it is possible to change the controller parameters of the ECMS controller. The reference value can be set at a high level when cruising at low speeds, to be able to accelerate with maximum power. The reference value can be lowered at accelerations to deplete the UC while the FC is working in a more efficient area. In the same way it is possible the change the equivalence factor to these two driving modes, making the UC power more “expensive” for cruising and “cheaper” for accelerating.
### List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_f$</td>
<td>Frontal area</td>
<td>[m²]</td>
</tr>
<tr>
<td>$A_{FC}$</td>
<td>Active surface area Fuel Cell</td>
<td>[m²]</td>
</tr>
<tr>
<td>$C_{comp}$</td>
<td>Constant compressor losses</td>
<td>[-]</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Air drag coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$c_{Fw}$</td>
<td>Contribution wheel inertia</td>
<td>[%]</td>
</tr>
<tr>
<td>$C_{P, air}$</td>
<td>Specific heat air</td>
<td>[J/(mol.K)]</td>
</tr>
<tr>
<td>$CP_{FC}$</td>
<td>Estimated cost price Fuel Cell</td>
<td>[€]</td>
</tr>
<tr>
<td>$CP_{UC}$</td>
<td>Estimated cost price Ultra Capacitor</td>
<td>[€]</td>
</tr>
<tr>
<td>$c_r$</td>
<td>Rolling resistance coefficient</td>
<td>[-]</td>
</tr>
<tr>
<td>$C_{tot,par}$</td>
<td>Total parallel Ultra Capacitors capacity</td>
<td>[F]</td>
</tr>
<tr>
<td>$C_{tot,ser}$</td>
<td>Total series Ultra Capacitors capacity</td>
<td>[F]</td>
</tr>
<tr>
<td>$C_{UC}$</td>
<td>Ultra Capacitor capacity</td>
<td>[F]</td>
</tr>
<tr>
<td>$dv$</td>
<td>Vehicle acceleration</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$E_{UC}$</td>
<td>Ultra Capacitor energy</td>
<td>[J]</td>
</tr>
<tr>
<td>$F_{acc}$</td>
<td>Acceleration force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_{air}$</td>
<td>Air resistance force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_r$</td>
<td>Rolling resistance force</td>
<td>[N]</td>
</tr>
<tr>
<td>$F_i$</td>
<td>Total drag force</td>
<td>[N]</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>$LHV_H2$</td>
<td>Lower heating value hydrogen</td>
<td>[J/kmol]</td>
</tr>
<tr>
<td>$I$</td>
<td>Current</td>
<td>[A]</td>
</tr>
<tr>
<td>$I_{air}$</td>
<td>Air-hydrogen ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$i_{FC}$</td>
<td>Current density</td>
<td>[A/m²]</td>
</tr>
<tr>
<td>$J$</td>
<td>Cost function</td>
<td>[W]</td>
</tr>
<tr>
<td>$M_{air}$</td>
<td>Molecular weight air</td>
<td>[kg/kmol]</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Hydrogen mass flow rate</td>
<td>[m³/s]</td>
</tr>
<tr>
<td>$m_v$</td>
<td>Vehicle mass</td>
<td>[kg]</td>
</tr>
<tr>
<td>$m_{subsystem}$</td>
<td>Ultra Capacitor subsystem mass</td>
<td>[kg]</td>
</tr>
<tr>
<td>$N_{FC}$</td>
<td>Number of Fuel Cells in stack</td>
<td>[-]</td>
</tr>
<tr>
<td>$N_{UC,par}$</td>
<td>Number of Ultra Capacitors parallel</td>
<td>[-]</td>
</tr>
<tr>
<td>$N_{UC,ser}$</td>
<td>Number of Ultra Capacitors in series</td>
<td>[-]</td>
</tr>
<tr>
<td>$P$</td>
<td>Power</td>
<td>[W]</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability function</td>
<td>[-]</td>
</tr>
<tr>
<td>$P_{ask}$</td>
<td>Power demand</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{aux}$</td>
<td>Auxiliary power</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{comp}$</td>
<td>Power demand from Fuel Cell air compressor</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{EM}$</td>
<td>Electric machine power</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{FC, max}$</td>
<td>Maximum power Fuel Cell</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{FC}$</td>
<td>Fuel Cell power delivery</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{H2}$</td>
<td>Hydrogen power</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{idle}$</td>
<td>Idle power Fuel Cell</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{intern}$</td>
<td>Internal power Fuel Cell</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>Maximum power</td>
<td>[kW]</td>
</tr>
<tr>
<td>$P_{UC, loss}$</td>
<td>Loss power Ultra Capacitor</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{UC, max}$</td>
<td>Maximum power Ultra Capacitor</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{UC}$</td>
<td>Upper Capacitor power</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{eq}$</td>
<td>Equivalent Ultra Capacitor power</td>
<td>[W]</td>
</tr>
<tr>
<td>$P_{UC}^{o}$</td>
<td>Optimal momentary set-point</td>
<td>[W]</td>
</tr>
</tbody>
</table>
### List of Symbols

- **Q**: Charge  
  - Unit: [C]

- **RUC**: Ultra Capacitor resistance  
  - Unit: [Ohm]

- **Rw**: Wheel radius  
  - Unit: [m]

- **s**: Equivalence factor  
  - Unit: [-]

- **T**: Torque  
  - Unit: [Nm]

- **to**: Estimated acceleration time  
  - Unit: [s]

- **t**: Time  
  - Unit: [s]

- **Ta**: Air temperature  
  - Unit: [K]

- **Tw**: Wheel torque  
  - Unit: [Nm]

- **U**: Voltage  
  - Unit: [V]

- **v**: Cell voltage  
  - Unit: [V]

- **v**: Vehicle speed  
  - Unit: [m/s]

- **VFC**: Internal Fuel Cell voltage  
  - Unit: [V]

- **VFC,tot**: Total Fuel Cell stack voltage  
  - Unit: [V]

- **Vth**: Theoretical Fuel Cell voltage  
  - Unit: [V]

- **∆hair**: Enthalpy difference  
  - Unit: [J]

- **ηc**: Air compressor efficiency  
  - Unit: [-]

- **ηEM**: Compressor electromotor efficiency  
  - Unit: [-]

- **ηFC**: Fuel Cell efficiency  
  - Unit: [-]

- **κair**: Air isentropic efficiency  
  - Unit: [-]

- **Πc**: Air compression ratio  
  - Unit: [-]

- **ρair**: Air density  
  - Unit: [kg/m³]

- **ρFC**: Mass density Fuel Cell  
  - Unit: [W/kg]

- **ρEUC**: Energy density Ultra Capacitor  
  - Unit: [Wh/kg]

- **ρPUC**: Power density Ultra Capacitor  
  - Unit: [W/kg]

- **ω**: Rotational speed  
  - Unit: [rad/s]
Literature

<http://www.aepint.nl>


Appendix

In this Appendix, the remaining figures for the different power train configurations from 5.2.1 are presented.

Figure A.1: Results on NEDC for 40 [kW] FC + 54.34 [F] UC

Figure A.2: Results on NEDC for 40 [kW] FC + 81.51[F] UC
Figure A.3: Results on NEDC for 40 [kW] FC + 108.68 [F] UC

Figure A.4: Results on NEDC for 40 [kW] FC + 135.85 [F] UC
Figure A.5: Results on NEDC for 45 [kW] FC + 54.34 [F] UC

Figure A.6: Results on NEDC for 45 [kW] FC + 81.51 [F] UC
Figure A.7: Results on NEDC for 45 [kW] FC + 108.68 [F] UC

Figure A.8: Results on NEDC for 45 [kW] FC + 135.85 [F] UC
Figure A.9: Results on NEDC for 50 [kW] FC + 54.34 [F] UC

Figure A.10: Results on NEDC for 50 [kW] FC + 81.51 [F] UC
Figure A.11: Results on NEDC for 50 [kW] FC + 108.68 [F] UC

Figure A.12: Results on NEDC for 50 [kW] FC + 135.85[ F] UC