SHIFT DYNAMICS MODELING FOR OPTIMIZATION VARIATOR SLIP CONTROL IN A CONTINUOUSLY VARIABLE TRANSMISSION

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ABSTRACT - V-belt type Continuously Variable Transmissions (CVT) are applied in an increasing number of vehicles as a result of their unparalleled shift comfort. Large ratio coverage allows for reduced engine speed, improved highway driving comfort and reduced fuel consumption. With the advent of the competing automatic transmissions with 6 or even 7 steps, it becomes increasingly important to further improve the performance in terms of efficiency, robustness and torque capacity of the CVT. Improvements on the efficiency of the push-belt CVT by reducing variator clamping forces to minimum values are well established. By reducing clamping such that the variator operates in its most efficient point, the mechanical load on this variator is minimized and hydraulic actuation losses are reduced. The control technique allows for best possible transmission efficiency, combined with improved robustness for slip damage.

In earlier studies, the relation between variator slip and functional transmission properties has been described. Conditions for optimum performance regarding efficiency and robustness were identified and validated for steady state conditions. However, stability robustness during ratio changes proved to be insufficient during tests in an experimental vehicle.

To deal with CVT slip dynamics during transient behaviour, a theoretical model is necessary. In this paper, a novel CVT shifting model, recently proposed by Carbone, is tested experimentally and compared to the model of Ide. The relationship between the clamping forces acting on the moving pulley sheave, the rate of change of speed ratio, the loading conditions and the belt velocity, are investigated both from a theoretical and an experimental point of view.

TECHNICAL PAPER - The low efficiency in present production CVTs is to a large extent caused by high clamping force levels. To prevent major slip events between pulleys and belt, the clamping forces are much higher than necessary for proper operation. These higher forces result in higher torque losses in the variator. Additionally, higher clamping forces require higher hydraulic pressures, thereby leading to increased pumping losses. Previous studies have shown that reducing these clamping forces result in a remarkable increase in efficiency (2). However, the risk of belt slip is increased, using these low clamping forces. Recent work by Van Drogen and Van der Laan (1) has shown that belt slip is allowed to a certain extend. Bonsen et al. (2) have demonstrated the possible efficiency gain by using slip control instead of the conventional control strategy.

This paper has two goals. The first goal is to identify the theoretical model, giving the best prediction of the CVT’s dynamical behavior during shifting maneuvers, for given values of the applied clamping forces, torque load and pulley angular velocity. The second goal is to propose a variator control strategy, which achieves increased robustness, efficiency and drivability compared to the control strategy proposed by Bonsen et al. (2).
VARIATOR SLIP

The theory of the slip controller is based on the power transfer in the CVT, which is due to the friction between pulleys and push-belt. For a V-belt variator, the relation between secondary clamping force $F_s$ and input torque $T_p$ can be represented by the slip dependent effective friction coefficient

$$\mu_{\text{eff}}(\nu) = \frac{T_p \cos \beta}{2F_s R_p}$$

(1)

where $R_p$ denotes the running radius of belt on the primary pulley and $\beta$ denotes the pulley wedge angle. It is assumed Equation 1 holds for both steady state and transient behaviour. Experiments have shown that $\mu_{\text{eff}}$ strongly depends on the CVT ratio and the amount of slip $\nu$ between belt and pulleys, but only weakly on clamping force and shaft speed as shown in the upper part of Figure 1. The traction increases linearly with slip until a maximum is reached at 1 [%] to 3 [%]. At higher slip levels, the traction decreases slowly with increasing slip. A distinction between a stable micro-slip area and an unstable macro-slip area can be made. The turning point between these regimes is at the highest possible effective friction coefficient. Close to this turning point also the maximum variator efficiency can be reached, as shown in the lower part of Figure 1.

![Figure 1: Effective friction coefficient $\mu_{\text{eff}}$ and efficiency $\eta$ versus slip $\nu$ measured at input speed of 300 [rad/s] for variator ratios low (0.43), medium (1.0) and overdrive (2.25).](image)

In this study the relative slip $\nu$ is defined as

$$\nu = 1 - \frac{r_s}{r_{s0}}$$

(2)

where $r_s$ is the speed ratio and $r_{s0}$ the speed ratio at no load conditions, assuming that there is no slip when the secondary shaft is unloaded, i.e. $T_s = 0$. The speed ratio $r_s$ is defined as

$$r_s = \frac{\omega_s}{\omega_p}$$

(3)

where $\omega_p$ and $\omega_s$ represent the primary and secondary rotational pulley speed respectively. Assuming the belt runs on perfectly circular paths on the wrapped angles of both pulleys, the geometric ratio $r_g$ can be defined by

$$r_g = \frac{R_p}{R_s}$$

(4)

with $R_p$ and $R_s$ the primary and secondary running radius, respectively. Since the variator is assumed to be slip free when $T_s = 0$, the continuity relation $R_p \omega_p = R_s \omega_s$ leads to $r_{s0} = r_g$. Since the variator is assumed to be slip free when $T_s = 0$, the continuity relation $R_p \omega_p = R_s \omega_s$ leads to $r_{s0} = r_g$. 
As shown in Figure 1 there is an optimum of the variator efficiency depending on the amount of slip. A safety factor can be defined that gives an indication of excess clamping forces, which is equal to 1 where the effective friction coefficient reaches its maximum.

\[
S_f = \frac{\text{max}(\mu_{eff})}{\mu_{eff}(v)}
\]

where \(\text{max}(\mu_{eff})\) represents the maximum value of \(\mu_{eff}\) over the whole range of positive values of \(v\) in a certain ratio. In a normal production CVT, an overclamping factor \(S_f\) of 3 up to 5 is not uncommon. In order to increase efficiency it is preferable to control the amount of slip, such that \(S_f \approx 1\). As shown in Figure 1 an efficiency gain of up to 15 [%] can be reached, when \(S_f\) is decreased.

At slip values beyond the point where the slope of \(\mu_{eff}(v)\) is zero, i.e. \(S_f = 1\), the micro slip regime changes to the macro slip regime as indicated by Bonsen et al. (2). This macro slip area is unstable, but limited excursions into this area may be allowed for push belt variators.

**TRANSIENT PUSH-BELT VARIATOR MODELS**

Transient variator models describe the relation between the CVT shift speed \(\dot{r}_p\) and clamping forces in the variator. Since the shift speed in the Jatco CK2-CVT is limited, only models that handle creep mode transient behaviour, Ide (3) and CMM (4), are evaluated on best agreement to reality. In all models the pulley thrust ratio is necessary for calculating shift forces.

**Pulley thrust ratio**

In a push belt CVT, the change in primary clamping force induces a shifting action. Here, the pulley thrust ratio \(\Psi\) at steady state is defined by

\[
\Psi(r_p, \tau) = \frac{F_{p}^*}{F_s}
\]

where \(F_{p}^*\) denotes the primary clamping force needed to maintain stationary CVT speed ratio and \(F_s\) the secondary clamping force.

Figure 2: Loaded \(\Psi\) measurements of the CK2-CVT at \(F_s = 10\) [kN] compared to the theoretical values for \(\Psi\) derived from the CMM model.

This pulley thrust ratio is dependent on the geometric ratio \(r_p\) and torque ratio \(\tau\). Here the torque ratio is interpreted as the transmitted torque to the torque that maximally can be transmitted by the given clamping force.
\[ \tau = \frac{1}{S_r} \]  

The pulley thrust ratio \( \Psi \), measured at a CK2 CVT transmission under different operating conditions, is depicted in Figure 2. This figure also shows the estimated \( \Psi \) by Carbone, further explained in the validation section.

**Ide model**

The Ide model is based on experimental results (3). This model can be expressed as

\[ \dot{r}_g = \omega_p k_i(r_g) \left( F_p - \Psi F_s \right) \]  

where \( k_i(r_g) \) is an experimentally obtained constant, which depends on the geometric ratio \( r_g \).

This equation relates the shift speed to the shift force \( F_p - F_s \Psi \) and input speed \( \omega_p \).

**CMM model**

In contrast to Ide’s model, the CMM model (4, 5) originates from a theoretical investigation. In creep mode, close to the stationary operation point, a linear dependency between shift speed and logarithm of the thrust ratio can be described by

\[ \dot{r}_g = \omega_p \Delta \beta \left( r_g \right) \ln \left( \frac{F_p}{F_s} \right) - \ln \Psi \]  

where \( \Delta \beta = \max|\beta' - \beta| \) is the maximum amplitude of the wedge half-angle variations along the contact arc. The groove angle is not constant along the wrapped arc because of the pulley bending due to elastic deformation and clearance in the bearings. \( \beta' \) is the non-uniform wedge half-angle of the deformed pulley, whereas \( \beta \) is the wedge half-angle of the undeformed pulley. The difference \( \beta' - \beta \) is described on the basis of the Sattler’s model (6). \( k_i(r_g) \) is a known function calculated using the theoretical model. The quantity \( \Delta \beta \) depends on the clamping forces acting on the two pulleys always being of the order \( \sim 10^{-3} \) [rad]. Equation (9) shows that creep-mode shifting takes place only thanks to bending of the pulley sheaves.

**Validations and comparison of Ide and CMM model**

Figure 2 shows the measured thrust ratio values compared with the prediction of the CMM model. It is clearly shown that when the torque ratio \( \tau \) is small, there is relatively significant disagreement between the theoretical predictions and the experimental outcomes. Nonetheless it has been found that in the case of used belts a very good agreement occurs also at low torque levels (5). This may support the idea that band-segments interaction, not considered by the CMM model, may have a key role of at small torque ratios. Thus, further investigation should be carried out. However, since the slip control strategy needs relatively high torque ratio values, this allows the use of the CMM model in next sections.

Figure 3 shows the shifting speed as a function of the logarithm of the thrust ratio for zero torque load and different values of instantaneous speed ratio \( r_g \). The figure shows a very good agreement between theory and experiment. At higher clamping force levels, the curves also fit well the measured data, which all fall on a straight-line. Data are matched by calculating the pulley bending parameter \( \Delta \beta \) which depends on the clamping force \( F_r \). A good agreement is also observed when the input speed \( \omega_p \) doubled, thus confirming the prediction of a direct proportional relation between input speed and shifting speed.
Figure 3: Validation at (a) $F_s = 20$ [kN] and $\omega_p = 1000$ [rpm], at (b) higher clamping force ($F_s = 30$ [kN]) with more elastic pulleys bending and (c) linear dependency of the input speed, $(\omega_p = 2000$ [rpm])

Ide gives indeed a linear relation between the shifting speed and the shift force ratio $F_p/F_s$ instead of the $\ln(F_p/F_s)$. Thus, if the Ide relation were really accurate it is expected the experimental results should follow a straight-line when plotted against the shift force ratio $F_p/F_s$. However Figure 4 shows this not to happen especially at low ratios $r_g$ where the curves significantly deviate from the straight line.

Figure 4: The measured shifting speed $dr_g/dt$ as a function of the shifting force $F_p/F_s$, $F_s = 20$ [kN] and $\omega_p = 1000$ [rpm]. A non linear behaviour is clearly visible, mainly at small ratios.

On the basis of the aforementioned considerations we conclude that the CMM model should be preferred to other models to describe the mechanical behaviour of V-belt CVT variators. Thus, this model will be used for modelling CVT variator dynamics.

MODELING SYSTEM DYNAMICS

In contrast to the model in the previous work of Bonsen et al. (2), both geometric ratio $r_g$ and relative belt slip $\nu$ are controlled in the optimized controller. Next the variator and actuation dynamics are derived, as shown in Figure 5, and interaction is analyzed.

Figure 5: Complete plant with variator dynamics $H_{var}$ and actuation dynamics $H_{LP}$ and $H_{PP}$
Variator dynamics

Analysis by Bonsen et al. (2) shows that the assumption of geometric dynamics being much slower than slip dynamics does not hold for fast shifting events. Slip dynamics are represented by

$$\dot{v} = \frac{r_s \dot{r} - r_g \dot{r}_g}{r_g}$$  \hspace{1cm} (10)

with

$$\dot{r}_s = \frac{\omega_p \dot{\omega}_s - \omega_s \dot{\omega}_p}{\omega_p^2}$$  \hspace{1cm} (11)

and $\dot{r}_g$ represented by the CMM model.

The CVT variator dynamics shown in Figure 6 can be described by

$$\dot{\omega}_p = \frac{T_e - T_p}{J_e} \hspace{1cm} \dot{\omega}_s = \frac{T_s - T_d}{J_s}$$  \hspace{1cm} (12)

The torques generated on both shafts of the variator are described, based on equation (1), by

$$T_{p,s} = \frac{2F_\mu(v)R_{p,s}}{\cos \beta}$$  \hspace{1cm} (13)

In this description torque losses are neglected

The friction coefficient used in this model is described by

$$\mu(v) = k_0 v + k_2 v$$  \hspace{1cm} (14)

which is a linear approximation of the results shown in Figure 1. The micro slip regime is denoted by $i = 1$, whereas $i = 2$ denotes the macro slip regime.

Actuation system dynamics

The line pressure is determined by the PWM-signal duty cycle used to control the solenoid. In this system the pressure is limited between 6.6 and 42 [bar] and can be assumed linear proportional to the duty cycle. The transfer function from duty cycle to secondary pressure was estimated from a FRF-measurement (2). This estimation resulted in a third order low pass filter with a cut-off frequency of 6 [Hz] that holds for all operation points. The linearized transfer function $H_{LP}$ from duty cycle to secondary clamping force is obtained by multiplying the FRF-estimation with secondary cylinder surface $A_s$.

The shifting actuation model is visualized in Figure 7. Shifting in the CK2 is controlled by the stepper motor, regulating a primary clamping force differing from the steady-state force. The mechanical feedback attached to the moveable primary pulley sheave, moves the valve back to its equilibrium position, when the desired position is reached. The total primary pressure
consists of the pressure change due to valve operation and an external force on the cylinder originated from the secondary pressure passed via the push belt.

The transfer function $H_{pp}$ from stepper motor input $u_{\text{step}}$ to quantity $\Delta_{\ln F}$ is derived from this model. In case of the CMM model the term $\Delta_{\ln F}$ represents the logarithm of the shift force ratio minus the logarithm of the pulley thrust ratio.

$$
\Delta_{\ln F} = \ln \frac{F_p}{F_s} - \ln \Psi
$$

(15)

**Complete plant**

Using equations (8) and (10), rewriting geometric ratio $r_g$ to axial primary pulley position $x_p$, and defining the state vector $x = [v \ x_p]^T$, input $u = [u_{\text{sol}} \ u_{\text{step}} \ T_c \ T_d]^T$ and output $y = x$, the dynamics can be described, when linearizing, by

$$
\dot{x} = Ax + Bu \\
y = Cx
$$

(16)

The derived linearized system, including actuation systems, will be used for controller design. The plant is shown for two operating points in Figure 8. As shown the $H_{11}$ term is inverse dependent on input speed $\omega_p$, while the magnitude of the other transfer function increases with the increase of this speed.
The model has 4 inputs, where only $u_{sol}$ and $u_{step}$ can be controlled on implementation. In the present setup of the hydraulic system the solenoid input $u_{sol}$ controls the slip. The stepper motor input $u_{step}$, controls the primary axial pulley position that is related to the geometric ratio. The input torque $T_e$ is controlled by the driver via the throttle pedal and the output torque $T_d$ is determined by the road conditions. The latter two can be regarded as disturbances on the system.

**Interaction analysis**

To analyse the degree of mutual influence for 2 x 2 systems, the interaction measure proposed by Rijnsdorp (7) is most widely used, defined as

$$\kappa = \frac{H_{12} H_{21}}{H_{11} H_{22}}$$  \hspace{1cm} (17)

For $\kappa < 0$ this leads to diagonal dominance of the plant. The relative gain array $\Lambda$, a measure of interaction for decentralized control is defined by

$$\Lambda(H) = H \times (H^{-1})^T = \begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix}$$  \hspace{1cm} (18)

with

$$\lambda_{11} = \frac{1}{1 - \kappa}$$  \hspace{1cm} (19)

For the modelled linearized system the values of $|\lambda_{11}|$ and $|\lambda_{12}| = |1 - \lambda_{11}|$ are plotted in Figure 9.

![Figure 9: Relative gain at different operation points](image)

As shown, at low input speeds, $\Lambda$ is near the identity matrix $I$. This implies small interaction between ratio and slip plant. At higher speeds diagonal dominance of the whole plant is still present, however more interaction is visible. This could be due to the proportional dependency of the input speed on the shift speed, while slip dynamics depend on the inverse of the primary speed. The transfer function from shift force to shift speed could than becomes relatively larger with respect to the transfer function from clamping force to slip, leading to more interaction.

**CONTROL STRATEGY AND IMPLEMENTATION**

Based on the results of the interaction analysis, i.e. limited influence of the off-diagonal terms, a decentralized controller for both ratio and slip is proposed, as represented in Figure 5. To take the interaction between the plant in- and outputs into account, sequential loop closing is applied. Since the ratio controller should always work during driving and the clamping...
force can also be applied in open-loop control, i.e. the slip controller can be switched; the ratio control loop is closed first. This ensures stability of the ratio controller.

![Figure 10: Block diagram representation of the decentralized control design with plant $H$ as represented in Figure 5.](image)

**Ratio control**

For optimal performance a PI controller is used for controlling the stepper motor.

$$u_{step} = \left( P_r + \frac{I}{s} \right) (x_{p,ref} - x_p) \quad (20)$$

The shifting process already exhibits a sufficient amount of damping. Therefore a differential action is not necessary, but a high controller gain can be applied. By manually tuning the controller an optimum in driveability and performance was found. For the setpoint $x_{p,ref}$ of this controller the variogram for the CK2 CVT is used (8).

**Slip control**

With the interaction coefficient $\kappa$ depicted in Equation (17), the equivalent for the slip plant $H_{11}$ can be described by

$$H_{11,eq} = H_{11} \left( 1 - \kappa H_{CL,22} \right) \quad (21)$$

with $H_{CL,22}$ defined as the closed loop transfer function of the ratio control. After closing the ratio control loop, the slip control loop can be closed using $H_{11,eq}$. At the crossover frequency $\kappa$ is close to zero, indicating small interaction.

In order to find a controller with stability of the closed loop system, good gain and phase margins, robustness with respect to unmodelled dynamics and optimal performance, LQG (Linear Quadratic Gaussian) control design is purposed. The slip plant is both observable and controllable in all operating points and therefore LQG can be applied. The state space notation of the slip plant should be extended with process noise $\xi$ and measurement noise $\theta$ that are uncorrelated Gaussian stochastic processes.

$$\dot{x} = Ax + Bu + \xi \quad y = Cx + \theta \quad (22)$$

LQG control is a combination of optimal state feedback and optimal state estimation. Therefore the state feedback matrix $K_r$ and the Kalman gain $K_f$ are obtained by minimizing the criteria

$$K_r = \arg\min E\left\{ x^T Q x + u^T R u \right\}$$

$$K_f = \arg\min E\left\{ (x - \hat{x})^T (x - \hat{x}) \right\} \quad (23)$$

Where $Q$ and $R$ are the weighting matrices for states and input respectively and $\hat{x}$ represents the optimal estimate for state $x$.

In standard LQG design an integral action is not included, therefore the slip plant is augmented with an integrator. Therefore in the slip plant the output equals $[v/s \, v]$. 

For $Q = qC^TC$, with $q = [100 10]$ and $R = 1$ are chosen. By using this term for $Q$ output $y$ is weighted rather than the state $x$. For the noise weighting matrices, covariances equal $\Xi = \mathbb{E}\{\xi\xi^T\} = \xi^2 I$, with $\xi = 1$ and $\Theta = \mathbb{E}\{\theta^2\} = 10^4$. This latter term is chosen this large, in order to keep control gains at high frequencies low, such that the influence of the large amount of measurement noise in automotive environment is reduced.

The slip plant is depending on many parameters, mainly on the difference between the micro and macro slip regime, but also on ratio $r_g$ and input speed $\omega_p$. Since the setpoint for the slip controller is on the turning point of both slip regimes and in the micro slip area the plant is stable, the macro slip regime is of main concern for robust stability. Assuming a friction coefficient independent of the amount of slip in the macro slip area, the plant is marginally stable at that point.

The worst case plant regarding robustness stability, disturbance and noise rejection was found at $\omega_p = 100$ [rad/s] and $r_g = 2.15$. This is chosen to be the nominal plant. The LQG control design was preformed on this plant. The open loop including the designed controller at this working point, has the smallest phase and gain margins (45.1 [deg] and 1.9, respectively), largest maximum sensitivity (2.3) and largest bandwidth (4.3 [Hz]). Robustness margins in other operation points are higher. However bandwidth decreases with respect to their optimal control design.

The input sensitivity of the system is shown in Figure 11. At higher input speeds the cross term sensitivity increases, but always remains under 0 [dB].

Compared to the gain scheduled PI controller proposed by Bonsen et al. (2), using overall plant performance and robustness of the slip control is increased. Especially when using the plant including ratio dynamics at 300 [rad/s], performance and robustness drops till poor values, even far over the required maximum sensitivity of 1.8.
VEHICLE IMPLEMENTATION RESULTS

For evaluation of the closed loop system, especially with regard to fuel consumption and robustness due to high and unpredictable load disturbances, the proposed controller is implemented in a production car. The test vehicle used in this study is a Nissan Primera 2.5i with Jatco CK2-CVT with a torque capacity of 250 [Nm]. The vehicle is tested on a chassis dyno, including a flywheel and eddy current brake to simulate road load conditions.

By measuring the axial position of the primary pulley with a linear displacement sensor, the ratio $r_{s0}$ under no load conditions can be estimated, while the two original Hall sensors provide the conventional ratio signal $r_s$. The system is accurate enough to detect slip with a resolution of 0.1 [%].

The controller produced the similar results at steady state behaviour as the PI-controller. Since the controller maintains the secondary clamping force at minimum level during steady state driving, increased efficiency level compared to the conventional controller is achieved. This efficiency gain is still bounded by the mechanically limited minimum clamping force level in the CK2-CVT.

A vehicle speed trajectory is followed to study the performance of the ratio controller, as shown in Figure 12. The trajectory was chosen to get insight during a common acceleration during driving. It shows that the steady state error is smaller than 0.1 [mm]. During fast shifting the error peaks to 1.5 [mm], due to the limited closed loop bandwidth.

Figure 12: Ratio tracking

Figure 13: Response to sudden accelerations. From 10 to 30 [km/h] in the left plot, from 30 to 60 [km/h] in the right plot.
The same trajectory is used to check disturbance rejection of the slip controller. During the sudden acceleration a torque peak is induced by the engine on the driveline. To achieve a good comparison between the gain-scheduled PI-controller and the developed LQG controller, feed forward was not used in this experiment. The slip setpoint was between 1 and 1.5 [%], depending on the ratio. Results are shown in Figure 13. Two slip events are shown, proving a faster reaction of the LQG-controller.

Although slip still exceeds 5 [%] using the LQG-controller, this can be reduced using feed forward control by estimating the required clamping force from the engine torque.

CONCLUSIONS AND RECOMMENDATIONS

The CVT variator model is extended with transient mode behaviour. For calculating shifting speed in this overall model, the CMM model gave best results. In future research more principle work on friction behaviour in the belt could be of interest, especially at low clamping force levels and relative high slip levels.

The developed slip controller based on the extended model, gives better disturbance attenuation compared to the previous controller, while the improved efficiency level is maintained. With this design the maximum performance with the present actuation hydraulics in the CK2-CVT is reached. Adapting this system for optimal control purposes is recommended. Still research is necessary to investigate long-term effects when using slip control with respect to variator damage.

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