Robust Sheet Control in an Uncertain Printer Paper Path

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The research described in this abstract focuses on sheet feedback control in an uncertain printer paper path. Given a prescribed sheet reference trajectory, $x_{r}$, the control goal is to make sheets track the reference trajectory in the presence of uncertain system parameters and to attenuate disturbances acting on the system. As a case-study we consider the three pinch paper path depicted in Fig. 1, in which each pinch is driven by a separate motor. The sheet position $x_{s}$, defined as the position of the leading edge, is assumed to be measured.

![Figure 1: Schematic representation of the printer paper path.](image)

The layout we adopt for the control design is an hierarchical structure consisting of low level motor control loops and a high level sheet control loop. For the controller synthesis, the motor control loops are assumed to be ideal and the focus is on the design of the high level control loop. Since we are dealing with a tracking problem, the model of the sheet flow is formulated in terms of its piecewise linear error dynamics:

$$\dot{q} = Fq + (G_i + \Delta G_i) \mu + V \nu$$

for $\{x_{r}, x_{s}\} \in \mathcal{I}$, $i \in \mathcal{I}$

$$z = Hq,$$

where the state vector $q$ is defined as $q = [e_s \ \dot{e}_s \ \dot{e}_s]^T$, with $e_s = x_{r} - x_{s}$ the tracking error. Furthermore, $\mu$ and $\nu$ represent the control input and disturbances, respectively. The uncertainties in the system parameters are captured in $\Delta G_i$. The partitioning of the state space is represented by $\{\mathcal{I}\}_{\mathcal{I} \subseteq \mathcal{R}}$, with $\mathcal{I} = \{1, 2, 3\}$ the index set of sheet regions. The upper bound of the uncertainty term $\Delta G_i$ is assumed to be known a priori: $|\Delta G_i| |\Delta G_i|^T \leq E_{G_i}E_{G_i}, i \in \mathcal{I}$, where $E_{G_i}$ is a constant matrix with the same dimensions as $\Delta G_i$. Given the model of the sheet flow, the goal is to find a feedback controller that stabilizes the error dynamics. The control law we propose is based on state feedback:

$$\mu = -Kq$$

Based on the results in [1], the controller gain $K = QP^{-1}$ can be calculated by solving the following LMIs:

$$0 < P = P^T$$

$$0 > \begin{bmatrix} \Omega_i & PH & QT \\ HP & -I & 0 \\ QT & 0 & -\epsilon I \end{bmatrix}, i \in \mathcal{I},$$

with

$$\Omega_i = PF^T + FP - QT^T G_i^T - G_i Q + \gamma^2 V_i^T \epsilon E_{G_i} E_{G_i}^T.$$  

The effectiveness of the control design approach has been demonstrated in practice on the experimental paper path setup, shown in Fig. 2. To show the robustness against uncertain system parameters, the actual transmission ratios between motors and pinches differ from the ones used in the model for the controller synthesis. Fig. 3 shows the sheet tracking error in case the reference trajectory is a ramp profile. It can be seen that after a transient behavior, the sheet error is indeed controlled towards zero.

![Figure 2: Photo of the experimental paper path setup.](image)

![Figure 3: Experimentally obtained sheet tracking error.](image)

References