Matlab Simulink modelling of a hybrid vehicle
CONTENTS

Introduction ................................................................................................................. 3
Chapter 1: Background .............................................................................................. 4
  1.1 FEV motorentechnik ......................................................................................... 4
  1.2 Project background ......................................................................................... 4
Chapter 2: Clutch ...................................................................................................... 6
  2.1 Basic clutch operation ..................................................................................... 6
  2.2 Clutch control ................................................................................................. 7
  2.3 Clutch modelling ............................................................................................. 8
Chapter 3: The gearbox .......................................................................................... 12
  3.1 Gearbox layout .............................................................................................. 12
  3.2 Ravigneaux set .............................................................................................. 14
  3.3 Calculating the dimensions of the gearbox parts .......................................... 16
  3.4 Gearbox model ............................................................................................. 17
Conclusion ................................................................................................................. 19
  Further improvements .......................................................................................... 19
Appendix A: Formula list ......................................................................................... 20
Appendix B: Entire Matlab Simulink model .......................................................... 21
Introduction

Due to tougher regulations on vehicle emissions and the increased fuel costs there is a search for more fuel-efficient vehicles. One solution to create such a vehicle is making it a hybrid. These vehicles consist of an internal combustion engine as well as an electric engine. More and more car companies are developing such a hybrid vehicle. There are multiple layouts possible, all with their own advantages and disadvantages. In this thesis a layout currently being constructed at FEV motorentechnik in Aachen is discussed. This vehicle will be modelled using Matlab Simulink. This model has to contain the detailed characteristics of each subcomponent of the vehicle. This model should be so that in the future it can be applied on other vehicles by simply changing the parameters and choosing the right subcomponents of the specific vehicle. The goal is to use this model in the future for software testing by SIL (software in the loop) and concept cars. This can save time and money compared to doing all calibration on the actual vehicle.
Chapter 1: Background

1.1 FEV motorentechnik

FEV is an internationally recognized leader in the design and development of internal combustion engines, and a major supplier of advanced testing and instrumentation products and services to some of the world's largest powertrain OEMs.

Founded in 1978 by Prof. Franz Pischinger, today the company employs a staff of over 1,300 research and development specialists on three continents.

Figure 1.1 FEV headquarters

1.2 Project background

Currently a hybrid SUV concept car is being built at FEV. The layout of this hybrid vehicle is shown in figure 1.2

As can be seen in the vehicle layout, the vehicle comprises of two power sources, namely an internal combustion engine (ICE) and a motor-generator (MG), which is an electric engine. The drivetrain furthermore consists of a dual mass flywheel (DMF), a wet plate clutch, a planetary gear set (PGS), a 6 gear automatic gearbox (AT) and differential (diff).

Figure 1.2 Layout of hybrid vehicle.
As can be seen there is no torque converter used in this layout. Normally when using an automatic gearbox one would also need a torque converter. In this case use is made of the geared neutral principle. Since you have a planetary gear set attached to the input side of the gearbox it is possible to calibrate the electric engine so that the input speed of the gearbox is zero even when the ICE is running. This principle is visualized in figure 1.3. In this figure the angled line represents the velocities and the vertical lines the torques in the corresponding points. There are a couple of advantages of eliminating the torque converter. First of all you save additional weight and create space by integrating the EM in the transmission and secondly a torque converter causes significant losses in some operating areas.

Figure 1.3 geared neutral principle

Using this vehicle layout multiple advantageous situations concerning fuel economy and performance can be achieved. The main functions are:
- Start-stop: switching off the ICE when stopping
- Boosting: producing extra power stored in the battery
- Regenerative braking: storing waste energy during braking into the battery
Chapter 2: Clutch

2.1 Basic clutch operation

A clutch is a device for coupling or uncoupling two parts of a shaft. In this vehicle layout a wet multi-plate clutch is used between the ICE and the MG as well as inside the automatic gearbox. In the gearbox planetary gear set configurations are exchanged by means of wet multi-plate clutches. These clutches consist of alternating layers steel and friction plates.

By applying hydraulic pressure to the pressure chamber, the piston displaces inside the cylinder. During the displacement from \( s=0 \) until \( s_{ap} \) the clutch plates aren’t yet in contact with each other. In this part of the displacement the hydraulic pressure compensates the friction in the cylinder as well as the spring force. From \( s_{ap} \) until \( s_{max} \) the clutch plates make contact and torque is being transferred. A model of a clutch is displayed in figure 2.1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{ap} )</td>
<td>Action point of the clutch. From this displacement upwards, there's a contact between the clutch discs.</td>
</tr>
<tr>
<td>( s_{max} )</td>
<td>End stop of the piston.</td>
</tr>
<tr>
<td>( A )</td>
<td>Effective surface of the piston.</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>Preload of Spring. Force on piston by spring at ( s=0 ).</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>Spring rate of piston pull-back spring.</td>
</tr>
<tr>
<td>( d_1 )</td>
<td>(Pseudo-)spring rate of the clutch discs. The force to compress the discs a tiny ( s ) is assumed to be ( s ) times ( d_1 ).</td>
</tr>
<tr>
<td>( K )</td>
<td>Hydraulic damper constant. Respectively internal friction of the hydraulic system. The friction is regarded as a force on the piston of ( v ) times ( k ) when it moves with velocity ( v ).</td>
</tr>
</tbody>
</table>

Figure 2.1 clutch model
2.2 Clutch control

As discussed above, the clutches are actuated by hydraulic pressure. For clutches the way this pressure is build up is an important parameter. The figure 2.2 shows a gearshift. The overall pressure build up can be seen as the blue line in figure 2.2. The gray line represents the torque. The black line represents the velocity. This pressure buildup is in general the same but has to be calibrated for each specific situation. The first part of the pressure buildup is the piston stroke phase. In this phase the piston moves from position \( s=0 \) to \( s_{up} \). During this phase no torque can be transmitted. Since this phase is actually lost time, it is favorable to keep it as short as possible. When this first pressure pulse is taken too large there isn’t a smooth torque transfer. This is not acceptable so it is important to calibrate this pulse to its optimum, which is on the one hand a fast response and on the other hand no torque overshoot. The second phase is the torque phase. During this phase the clutch is capable of transferring torque and the torque path changes from the “old” gear over the new. During this phase there isn’t yet a velocity drop. Next phase is the inertia phase. This is the phase that the angular velocity actually drops. This phase starts as soon as the torque capacity of the “new” or engaging clutch exceeds the current torque. In the last phase the pressure is maintained at its maximum. This is done to get the maximum torque transfer capacity, since the higher the pressure, the higher the transferable torque. The red line finally is the torque reduction request. This is a signal, which is send to the ICE to request a lower torque output. This is done to avoid a high torque peak during the inertia phase.

![Figure 2.2 pressure build up](image)

<table>
<thead>
<tr>
<th>Sol. control</th>
<th>Piston stroke</th>
<th>Torque phase</th>
<th>Inertia phase</th>
<th>Sol. control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Start</td>
<td>Shift Start</td>
<td>Shift end</td>
<td>Control end</td>
<td></td>
</tr>
</tbody>
</table>
2.3 Clutch modelling

The model of the clutch system has the inputs pressure and requested torque. The pressure is not given in bar but in current (A). This is chosen since the interface between the actual AT and the software is also in current (A). This is the current, which flows to the solenoid. The solenoid is an electronic valve. The current of the solenoid has a more or less linear relation to the applied pressure. The second input, the requested torque, is the torque on the engine side of the clutch. The outputs are the input and output velocities. These depend on the input torque and the applied pressure. For instance when the clutch has no applied pressure the output velocity is zero and when the applied pressure is large enough to fully close the clutch then the input velocity equals the output velocity. The third output of the clutch model is the lockup signal. This output goes to one when the clutch is fully closed and the input velocity equals the output velocity. In figure 2.3 the basic layout of the Simulink clutch model is displayed.

Figure 2.3 clutch model in Simulink

The torque capacity of the modeled clutch is determined with the formula 2-1

\[ T_c = n \cdot p \cdot A \cdot R \cdot \mu \cdot \text{sign}(\omega_1 - \omega_2) \]  

(2-1)

Where \( n \) = number of friction surfaces, \( p \) = effective pressure, \( A \) = pressure surface, \( R \) = effective radius, \( \mu \) = tangential friction coefficient and \( \omega \) = tangential speed

The effective pressure \( p \) is not the same as the applied pressure from the solenoid. The applied pressure has to be reduced by the friction pressure and the spring pressure to find the surface pressure. The friction pressure is the amount of pressure needed to overcome the friction between the clutch piston and cylinder. The spring pressure is the amount of pressure needed to compress the clutch springs.
The Simulink model of the clutch comprises of a couple of sub models as can be seen in figure 2.4. The first part is determining the surface pressure. Secondly the torque capacity is determined. And finally using the applied torque and the torque capacity, the input and out velocities are determined. Figure 2.5 shows the first part.

Figure 2.4 clutch sub model layout

Figure 2.5 clutch surface pressure model

The input of this submodel is the solenoid current. In a look up table the corresponding applied pressure is determined. Next the flow rate is determined. Integrating the flow rate gives the volume into the cylinder. This volume first fills the clearance volume, which covers all gaps and clearances. When the entire clearance volume is filled, the cylinder gets filled which displaces the piston. The piston displacement is the volume of oil in the cylinder devided by the piston surface. The piston force comprises of three contributions namely the spring force, the friction force and the clutch plate compression force. The spring force is determined using the piston displacement and formula 2-2. Using the same formula the clutch plate compression force is determined. The difference is that you don’t use the piston displacement but the compression of the clutch plates and of course a totally different spring coefficient. The third contribution to the piston force is the friction force and the spring preload force. These are both taken as constants. Finally dividing the piston force by the surface gives the clutch pressure.
\[ F_{spring} = c \cdot u \quad \text{Where } c = \text{spring coefficient and } u = \text{displacement} \quad (2-2) \]

This clutch pressure is used as the input for the second subsystem for determining the torque capacity. This subsystem is shown in figure 2.6.

![Clutch torque capacity sub model](image)

Figure 2.6 clutch torque capacity sub model

The torque capacity is determined using the already mentioned formula 2-1

\[ T_c = n \cdot p \cdot A \cdot R \cdot \mu \cdot \text{sign}(\omega_1 - \omega_2) \quad (2-1) \]

Next the requested torque is compared to the torque capacity. If the torque capacity is larger than the requested torque then the output torque equals the requested torque. But if the torque capacity is smaller than the requested torque then the output torque equals the torque capacity. If you know the torque on the input side and the torque on the output side then the input and output velocities can be calculated. This is done with the use with formula 2.2. In this formula in input velocity is determined using the input and output torque and the input inertia.

\[ \omega_{in} = \int \frac{T_{in} - T_{out}}{J_{in}} \quad (2.2) \]

![Torque capacity versus pressure](image)

Figure 2.8 clutch measurements versus model

Figure 2.8 shows the output torque on a simple ramp in the pressure. This is the same ramp as used in the measurement. This figure shows that the model output torque is almost exactly the same as
determined in the measurements. The maximum torque capacity is around 700 Nm at the maximum pressure of 17.5 bar.
3.1 gearbox layout

The gearbox is based on the lepelletier design. This means that the gearbox consists of two planetary gear sets. One normal planetary gear set and one ravigneaux set, which is a special planetary gear set, which will be discussed later. Furthermore it consists of five shift elements. The gearbox is shown in figure 3.1.

In each gear there are two shift elements active. This can be two clutches or a clutch and a brake. In table 3.2 the gear ratios and applied clutches or brakes in the corresponding gears are shown.

<table>
<thead>
<tr>
<th>Gear</th>
<th>Ratio</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>B1</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.53</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>2.14</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>1.48</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>1.16</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>0.87</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>0.68</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>REV</td>
<td>-3.09</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 3.2 gear ratio table
First of all a couple of important formulas are given of the important parts namely the planetary gear sets.

**Torque relationships of a simple planetary gear.**

\[ T_{q_r} = z \cdot T_{q_s} \]  \( \Rightarrow \) relation between ring wheel and sun wheel \hspace{5cm} (3-1)

\[ T_{q_c} = (z+1) \cdot T_{q_s} \]  \( \Rightarrow \) relation between sun wheel and planet carrier \hspace{5cm} (3-2)

\[ T_{q_r} = \frac{z}{z+1} \cdot T_{q_c} \]  \( \Rightarrow \) relation between ring wheel and planet carrier \hspace{5cm} (3-3)

**Torque relationships of a double pinion planetary gear.**

\[ T_{r} = z \cdot T_{s} \]  \( \Rightarrow \) relation between ring and sun \hspace{5cm} (3-4)

\[ T_{c} = (z-1) \cdot T_{s} \]  \( \Rightarrow \) relation between sun and planet carrier \hspace{5cm} (3-5)

\[ T_{r} = \frac{z}{z-1} \cdot T_{c} \]  \( \Rightarrow \) relation between ring and planet carrier \hspace{5cm} (3-6)

With  \( T_{q_r} \) the torque on the ring wheel.

\( T_{q_c} \) the torque on the planet carrier.

\( T_{q_s} \) the torque on the sun wheel.

\( z = \frac{\text{nr of teeth of ring wheel}}{\text{nr of teeth of sun wheel}}. \)

**General angular velocity equation**

\[ \omega_r + z \cdot \omega_r - (1 + z) \cdot \omega_c = 0 \]  Normal planetary gear set \hspace{5cm} (3-7)

\[ -\omega_s + z \cdot \omega_r - (z - 1) \cdot \omega_p = 0 \]  Double pinion planetary gear set \hspace{5cm} (3-8)

With  \( w_r \) the angular velocity of the ring wheel.

\( w_c \) the angular velocity of the planet carrier.

\( w_s \) the angular velocity of the sun wheel.

\( z = \frac{\text{nr of teeth of ring wheel}}{\text{nr of teeth of sun wheel}}. \)
3.2 Ravigneaux set

A ravigneaux set is a very compact planetary gear set that can provide multiple practical gear ratios. It consists of one planet carrier, one ring wheel and two sun wheels. In fact it consists of a normal and a double pinion planetary gear set. With both ring wheels and planet carriers coupled. This set has 4 degrees of freedom, namely the ring wheel, planet carrier and two sun wheels. This can be seen in figure 3.3a. With a ravigneaux set it is possible to realize a 4 gear automatic gearbox with reverse. If coupled with an additional planetary gear set one can realize the 6 gear automatic gearbox with reverse considered in this project. This gear set is also known as lepelletier gear set, first used in ZF’s 6HP26 transmission.

![Figure 3.3a ravigneaux set](image1)

![Figure 3.3b simple planetary gear set](image2)

In case of a ravigneaux set you have a set of two coupled equations

\[(z_s - 1) \cdot \omega_c = z_s \cdot \omega_r - \omega_{s\_small} \quad (3-9)\]
\[(z_l + 1) \cdot \omega_c = z_l \cdot \omega_r + \omega_{s\_large} \quad (3-10)\]

In these equations \(z_s\) means the ratio between the ring and the small sun and \(z_l\) is the ratio between the ring and the large sun.

Since the ravigneaux set consists of a coupled normal and double pinion planetary gear set, visualizing the torques and angular velocities of the ravigneaux set one needs to combine figure 3.4a and 3.4b.

The gearbox output torque is the torque on the ring wheel of the ravigneaux set. Using figure XXX balance of momentum you can visualize the torque split in each gear. The sum of the momentum in the ravigneaux set equals zero. So depending on the gear and the gearbox output torque all torque splits can be determined.

![Figure 3.4a normal PGS](image3)

![Figure 3.4b double pinion PGS](image4)

Since a ravigneaux set consists of a normal PGS and a double pinion PGS with coupled ring wheels and coupled planet carriers one needs to combine figure 3.4a and 3.4b in order to construct the balance of momentum for a ravigneaux set. This means overlaying the carriers and ring wheels. Since the distance between C and R isn’t the same in both cases what it needs to be, you choose them the same and compensate this by also changing one distance between S and R. Doing this gives figure 3.5.
Where Sl is the large sun, or the sun of the normal PGS and Ss is the small sun, or the sun of the double pinion PGS.

The diagram just created can also be used to represent the angular velocities of all components. This can be seen in figure 3.6. Here it can be seen that when two velocities are known the other two can be found.
3.3 Calculating the dimensions of the gearbox parts

Combining figure 3.1, table 3.2 and the angular velocity equations it is possible to derive the teeth numbers of the individual gears of the planetary gear sets.

The dimensions of the small sun and the ring of the ravigneaux set are already known since this gear set is also used as the PGS combining the ICE and MG at the transmission input side. By dismantling this part it is determined that the small sun has 28 teeth and the ring has 67 teeth.

The simplest gear is third gear. In this case clutch 2 and clutch 3 are closed this means that the small and the large sun of the ravigneaux set have the same angular velocity. Inserting this into equation (1) and (2), you find that the ring velocity or gearbox_out velocity is equal to the sun’s velocity. So PGS 1 then determines the entire gear ratio. And thus the ratio of PGS 1 is 1.48. The formula for the ratio of a normal planetary gear set with fixed sun is \( i = 1 + \frac{Z_s}{Z_r} \), with \( Z_s \) the number of teeth of the sun and \( Z_r \) the number of teeth of the ring. We assume \( Z_r = 68 \) then \( Z_s = 46 \).

The last dimension you need is the number of teeth of the large sun in the ravigneaux set. Now looking at the reverse gear and assuming the gearbox input speed 100. The angular velocity of the large sun is \( 100/1.48 = 67.6 \) and ring speed is \( 100/-3.09 = -32.4 \). Brake 2 is applied so the speed of the carrier is zero. Inserting this data into equation (2) you find \( Z_l \) is 2.08. The formula for the ratio with fixed carrier is \( i = \frac{Z_r}{Z_s} \). We know \( Z_r = 67 \) then \( Z_s = 32 \).

Summarizing the dimensions of the important parts are:

<table>
<thead>
<tr>
<th>Gear</th>
<th>Sun PGS 1</th>
<th>Ring PGS 1</th>
<th>Small sun ravigneaux set</th>
<th>Large sun ravigneaux set</th>
<th>Ring ravigneaux set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>46</td>
<td>68</td>
<td>28</td>
<td>32</td>
<td>67</td>
</tr>
</tbody>
</table>

Table 3.7 gearbox part dimensions
3.4 Gearbox model

A gearbox can be modeled in many different ways. It is chosen to model the gearbox in considerable depth. This means modeling each individual clutch, brake and planetary gear set. This makes it possible to do a part of the calibrating of the real gearbox using the model. This is normally largely done on the hardware of the gearbox on a test bench or in the car. This takes a lot of time because there are a lot of variables, which can be optimized. Using a model can reduce time and costs considerably.

Looking at the basic gearbox model layout in figure 3.8 one can see that the known inputs are the gearbox input and output velocity. Furthermore there are three additional inputs namely the demanded gear this is a value received from the HCU. The HCU (Hybrid Control Unit) is the vehicle controller, which for instance determines when a gearshift is required. Next is the transmission lock value. This determines whether the transmission is locked and acts like a brake on the input shaft. Locking the gearbox is done by activating all three clutches in the gearbox at the same time. This locking is used to start the internal combustion engine with the motor generator. Finally the last input is the neutral flag value. This value goes to 1 when gear lever is in neutral and is 0 in all other cases. The output values and thus the values that have to be determined are the gearbox input and output torque.

The gearbox model consists of multiple sub models. It contains three clutch sub models described above. Furthermore two brake models, which are basically clutches with the difference being that one side of the clutches has velocity zero. Besides these clutch models there is a sub model denoted as the clutch and brake pressure control. Finally there are the ravigneaux set sub model and a normal planetary gear set sub model.

The clutch and brake pressure control is build up so that first the requested gear is checked. In a table the clutch(es) and/or brake belonging to the requested gear ratio are checked. Next is applying the actual pressure build up to the concerning clutch(es) and/or brake. Furthermore at the same time the pressures of the clutch(es) and/or brake, of the previous gear, are decreased. The building and decreasing of the pressure has to be calibrated so that you get a smooth gear change.

A normal planetary gear set is described with the already mentioned formula 3-7.

\[
\omega_z + z \cdot \omega_r - (1 + z) \cdot \omega_c = 0
\]  

The first planetary gear set used in this gearbox is a special case since the sun gear is fixed to the gearbox housing and thus has \( \omega_s = 0 \).

This simplifies equation 3-7 into equation 3-12.

\[
z \cdot \omega_r - (1 + z) \cdot \omega_c = 0
\]  

Figure 3.8 basic layout gearbox model
Fixing one of the axes of a planetary gear set thus reduces the planetary gear set to a simple fixed gear ratio. The normal planetary gear set can in this case be modeled as a fixed ratio between input speed and output speed as well as between input torque and output torque.

The ravigneaux set has four degrees of freedom. These are the velocities of the ring wheel, planet carrier and two sun wheels. In each gear two velocities are known. The model of the ravigneaux set uses the two known velocities to determine the other two unknown velocities. The model layout can be seen in figure 3.9

![Ravigneaux set model layout](image)

Figure 3.9 Ravigneaux set model layout

The gearbox contains multiple torque splits. For instance in third gear, clutch 2 and clutch 3 are both activated. When clutch 2 and clutch 3 are both activated the angular velocity of both clutches is equal, but the torque applied on each of the clutches is not equal. The torque on clutch 2 plus the torque on clutch 3 equals the torque coming from the normal planetary gear set. It is important to know the torque applied to each clutch to find whether the clutch is open, slipping or closed. Therefore you have to calculate the torque splits in the entire gearbox at every time step.

All sub models have been integrated into the entire vehicle model, which can be found in appendix B.
Conclusion

Multiple Matlab Simulink models were created. First of all a clutch model. Secondly an automatic transmission of the lepelletier type. These submodels were integrated in a larger entire vehicle model. The submodels, clutch and gearbox can be used in other vehicle models simply by changing the parameters.

Further improvements

All inertias in the gearbox model have been estimated. These inertias can also be calculated when all component parameters are known. These can be found by taking the dimensions and weights of all components. With this data one can calculate its inertia. When the calibration of the concept car, which is currently being developed, is finished, the measurement data can be compared with the model results and the model can be fine-tuned.
Appendix A: Formula list

\[ T_c = n \cdot p \cdot A \cdot R \cdot \mu \cdot \text{sign}(\omega_1 - \omega_2) \]  \hspace{1cm} (2-1)

\[ \omega_{in} = \int \frac{T_{in} - T_{out}}{J_{in}} \]  \hspace{1cm} (2.2)

\[ T_{q_r} \approx z \cdot T_{q_s} \]  \Rightarrow relation between ringwheel and sunwheel  \hspace{1cm} (3-1)

\[ T_{q_c} = (z+1) \cdot T_{q_s} \]  \Rightarrow relation between sunwheel and planetcarrier  \hspace{1cm} (3-2)

\[ T_{q_r} = \frac{z}{z+1} \cdot T_{q_c} \]  \Rightarrow relation between ringwheel and planetcarrier  \hspace{1cm} (3-3)

\[ T_r = z \cdot T_s \]  \Rightarrow relation between ring and sun  \hspace{1cm} (3-4)

\[ T_c = (z-1) \cdot T_s \]  \Rightarrow relation between sun and carrier  \hspace{1cm} (3-5)

\[ T_r = \frac{z}{z-1} \cdot T_c \]  \Rightarrow relation between ring and carrier  \hspace{1cm} (3-6)

\[ \omega_r + z \cdot \omega_r - (1+z) \cdot \omega_c = 0 \]  \hspace{1cm} Normal planetary gear set  \hspace{1cm} (3-7)

\[ -\omega_r + z \cdot \omega_c - (z-1) \cdot \omega_c = 0 \]  \hspace{1cm} Double pinion planetary gear set  \hspace{1cm} (3-8)

\[ (z_s - 1) \cdot \omega_c = z_s \cdot \omega_r - \omega_{s\_small} \]  \hspace{1cm} (3-9)

\[ (z_i + 1) \cdot \omega_c = z_i \cdot \omega_r + \omega_{s\_large} \]  \hspace{1cm} (3-10)

\[ z \cdot \omega_r - (1+z) \cdot \omega_c = 0 \]  \hspace{1cm} (3-11)
Appendix B: Entire Matlab Simulink model