Measurement technique to determine modal parameters of friction induced resonance.

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Abstract

A frequency domain based measurement method is presented to determine modal parameters of friction-induced resonance in a mechanical system. This resonance occurs in the stick-phase and is due to the tangential stiffness in the friction contact in combination with the inertia of the system. The resonance frequency and damping are functions of the RMS level of the motion of the system and of the type of the applied excitation signal. Several experiments are carried out using random excitation and sine on random signals.

1 Introduction

In many mechanical positioning systems control performance is adversely affected by friction [1], [2]. The dynamical behaviour of these systems depends upon the state of the system. In the pre-sliding regime, the friction contact reacts as a stiffness [3]. In the sliding regime tangential stiffness diminishes and the damping behaviour becomes dominant.

Since a stiffness in combination with a mass or an inertia constitutes a second order system, a mechanical system with friction in the pre-slide phase will exhibit a resonance. The frequency and damping of this resonance will depend upon the relative movement in the friction contact. In [4] a measurement technique is described to estimate the apparent stiffness and damping in this phase. The results show an excitation signal level dependent magnitude of the bristle stiffness $\sigma_0$. The possible excitation level dependency of the damping magnitude being the sum of the bristle damping $\sigma_1$ and viscous damping $\sigma_2$ is not discussed. In this paper the results of [4] will be further extended.

2 Problem definition

2.1 Measurement of frequency response function in stick-phase

The aim of this research is to develop and test a measurement method that can be used to determine the Frequency Response Function (FRF) of a mechanical system in its stick-phase. From this FRF the frequency and damping of the resonance, caused by the friction induced stiffness and system inertia/mass, can be determined as a function of excitation signal characteristics.

2.2 Choice of excitation signal

Since friction behaviour depends heavily upon the state of the system, the spectral energy distribution of the excitation signal must be carefully chosen. A random noise excitation signal with known RMS value and bandwidth is often used for FRF measurements in non-linear systems, since the resulting FRF is the best linear approximation of the non linear system. [5]. However its gaussian amplitude distribution with a crestfactor of approximately 3.5 can make local linearization impossible and might force a system, subjected to friction, through the stick-phase into the slip-phase.
The amplitude distribution of sine on random is narrower but still has the benefit of linearization of non-linear behaviour. Figure 1 shows the amplitude distribution for a random signal and a sine on random signal, both with equal RMS value of 0.5V. The sine on random signal has a crestfactor of approximately $\sqrt{2}$, the crestfactor of the random signal is 3 or more.

![Figure 1: Amplitude distribution](image)

Both types of excitation will be used in the measurements to determine the possible influence of 1) the difference in crestfactor and 2) the sine frequency on the estimates of the resonance frequency and damping.

3 Measurements

3.1 Measurement set-up

The mechanical system consists of a 20 W electric DC collector motor with encoder (Parvex RS110MR1000) and is powered by a voltage to current converter (Philips DCPA 50/2). The system response can be measured in terms of angular displacement, velocity or acceleration. The expected rotations in the stick-phase are in the order of magnitude of $1 \times 10^{-5}$ rad. This will be difficult to measure with an angular position sensor. In acceleration terms however, $1 \times 10^{-5}$ rad at 100 Hz corresponds to 4 rad/s$^2$ and can be measured reliably with an angular acceleration sensor system [6] (Kistler type 8832, TAP8696 sensor and 5130A coupler).

The measurement set-up, see figure 2, is placed onto a granite base to reduce the influence of external vibrations.

![Figure 2: Measurement set-up](image)

Due to the brushes, seals and bearings in the motor the system is subjected to friction. In the LuGre model [3] this friction influence is modelled with $\sigma_0$, the bristle stiffness, $\sigma_1$, the bristle damping and $\sigma_2$, the viscous damping. In order to determine the influence of these parameters on the system dynamics, FRF's are measured. $V_{in}$ is the input signal into the linear voltage to current converter. Its output current $I$ is converted by the motor into a torque $T$. The output signal is the angular acceleration of the motor shaft $\dot{\theta}_{out}$ and is measured with a Kistler angular acceleration sensor. Both signals are processed with a SigLab 20-42 dynamic signal analyzer [7], and MatLab 5.3. A schematic representation of the system is given in figure 3.

![Figure 3: Schematic representation of system](image)
3.2 Measurement procedure

In all measurements the $H_1$ frequency response function i.e. $G_{AB}/G_{AA}$ was determined between $V_{in}$ and $\phi_{out}$ in the frequency range 0 Hz to 2000 Hz with a frequency line spacing of 1.25 Hz, resulting in an effective frequency resolution of 1.5*1.25 Hz due to Hanning weighting of both channels. Every FRF was calculated using linear averaging of 20 data blocks of 800msec, acquired with free run averaging.

Table 1 gives an overview over the applied excitation signals $V_{in}$. In all excitation signals, the random noise component is band-limited to 2000Hz. The sine frequencies are chosen to 1) coincide with an FFT line to avoid leakage, 2) not be harmonically related to 50Hz and 3) to enclose the friction-induced resonance because the relation between its modal parameters and the excitation frequency is unknown.

<table>
<thead>
<tr>
<th>Series</th>
<th>Excitation signal</th>
<th>Sine component</th>
<th>Noise component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Sine on random</td>
<td>190Hz 0mV→32.5mV</td>
<td>0.5mV</td>
</tr>
<tr>
<td>2</td>
<td>Sine on random</td>
<td>810Hz 0mV→210mV</td>
<td>0.5mV</td>
</tr>
<tr>
<td>3</td>
<td>Random</td>
<td>none</td>
<td>1mV→80mV</td>
</tr>
</tbody>
</table>

Table 1: Excitation signal characteristics

A 20dB attenuator (see fig. 3) was placed to increase the dynamic range of the reference channel due to its electrical noise floor and the very low measurement signals.

4 Results

4.1 Slip phase dynamics

A visible movement of the motor shaft required a $V_{in}$ of approximately 1V. In this measurement the 20dB attenuator was omitted. As a reference for subsequent measurements the FRF $\phi_{out}/V_{in}$ of the system operating in the slip-phase was measured, see figure 4. The structural resonance at 1600Hz is due to the inertia’s $J_1$ and $J_2$ and stiffness $C$. The phase-plot indicates also high frequency poles caused by the inductance of the motor and the amplifier.

4.2 Stick phase dynamics

To investigate the stick-phase dynamics over a wide excitation range, the excitation signal $V_{in}$ will be varied between 0.5mV and 80mV. The 20 dB attenuator is placed in the generator signal path.

4.2.1 Band limited random noise measurements

Figure 5 shows the FRF’s obtained after excitation with 45mV and 1mV. The friction-induced resonance at about 500 Hz for 1mV and about 400 Hz for the 45mV excitation is clearly visible and its modal parameters depend heavily upon excitation level. The frequency and damping of the second resonance near 1700 Hz also have changed compared to figure 4 due to an increased stiffness $\sigma_0$ and decreased damping $\sigma_1$. 

Figure 5: FRF of the system in the stick phase.
In figure 6 and 7 the measured modal parameters of the two resonances are given as function of the RMS value of the total angular acceleration level.

The results of the measurements are displayed in figures 8-11 relating the modal parameters to RMS rotation values calculated by double integration of the averaged angular acceleration power spectra.

Figure 6 shows a strong influence of the RMS level of the response signal on frequency and damping of the friction induced mode. The influence on the modal parameters of the second mode is much weaker as can be seen in figure 7.

**4.2.2 Sine on random measurements**

As mentioned in section 2.2 the sine on random excitation signal has a crestfactor of approximately $\sqrt{2}$. The possible influence of the crestfactor and sine frequency on the modal parameters is tested by the two series of measurements as defined in table 1.

Figure 8 shows that for comparable rotation levels, the resonance frequency estimates from the 810 Hz sine on random experiment are all higher than the estimates from the noise and 190 Hz sine on random experiments.

Figure 9 shows a strong difference between the damping estimates from the noise experiment and the results from the sine on random experiments.
0.6 0.8 1 1.2 1.4 1.6 1.8 2
0 0.5 1 1.5 2 2.5 3 3.5 4
Total angular displacement [rad RMS] x x 10^-5

Figure 10: Frequency of second resonance.

As in figure 8, the estimates for the frequency of the 1700 Hz resonance have a tendency to be higher than the estimates from the other two experiments.

Figure 11: Damping of second resonance.

Unlike in figure 9, figure 11 shows a fairly consistent estimate for the damping of the second resonance based on the noise experiment and the 190 Hz sine on random experiment. Both in figure 9 and 11, the damping estimates from the 810 Hz sine on random experiment are consistently lower than the results from the other experiments.

4.2.3 Reproducibility

Since the stiction behavior in the system is assumed to depend on the shaft angle, all measurements are done with the motor shaft in the same position. The three series of measurements show consistent results at low values of displacement. Reproducibility of the measurement results however was not tested.

5 Conclusions

This paper demonstrates a frequency domain measurement technique for determining the modal parameters of the friction-induced resonance of a system in its stick-phase. To determine these parameters, frequency response functions are measured between the input of a voltage to current converter and the angular acceleration of the current fed example system. The applied excitation signals are two sine on random signals with different sine frequency and one random noise signal.

Experiments show a significant influence of the type of excitation signal and its RMS value upon the frequency and damping estimates of the stiction induced resonance and the first structural resonance.

The reproducibility of the results is not tested and can the subject for further work.

References