Dynamics and Control of the Zero Inertia Powertrain

Due to rotating inertias within the engine and transmission, the response of a vehicle during large engine speed transients may appear reluctant or even counteracting. Reminiscent of comparable behaviour in aircraft jet-propulsion, this phenomenon is also referred to as ‘jet-start’. To overcome this behaviour, a CVT powertrain is augmented with a planetary gear set and a compact steel flywheel. The new transmission seamlessly combines two contradictory features: the driveability in terms of the pedal-to-wheel response is greatly improved and a large leap towards optimal fuel economy can be made. This is achieved by cruising the vehicle at extremely low engine speeds, using the large ratio-coverage of the CVT. The flywheel acts as a ‘peak shaver’ during speed shifts: it delivers power during engine acceleration and absorbs kinetic energy during engine decelerations. In this paper, system dynamics and control aspects of the new powertrain are discussed.

1 Introduction

Focus is on the dynamics and control of a powertrain incorporating a Continuously Variable Transmission (CVT), a power-splitting parallel stage and a flywheel. In the sequel this powertrain is referred to as ‘Zero Inertia’ (ZI) Powertrain. The flywheel is present to eliminate the inverse response known as jet-start behaviour, which is common in powertrains that are operated at low engine speeds for better fuel efficiency. In the paper, the new powertrain is described, after which the dynamical behaviour and the controlled system are analyzed. The model used for the dynamical analysis assumes the engine torque to be generated in some way not to be specified, but characterized by its frequency contents only. The model considered in the controller design is similar, although the torque converter spring and tire slip are omitted intentionally. Simulation results from three different control strategies conclude the paper. Currently, experiments on a prototype vehicle (see Figure 1) are conducted in order to show the improvements both in fuel economy and in driveability.

Figure 1: Zero Inertia prototype vehicle

2 System Description

Figure 2 shows a schematic drawing of ZI Powertrain. The powertrain comprises a 1.6 l 4 cylinder petrol engine, a commercially available metal pushbelt CVT including auxiliary systems such as a torque converter, a DNR-set, a final reduction gear and a differential. In parallel, a compact steel flywheel is connected to this CVT by a planetary gear set with additional gears but without additional clutches. The planetary gear set operates as a power-split device, deliberately splitting power of
the flywheel to and from primary and secondary side whenever the ratio of the CVT is changed.

The flywheel speed $\omega_f$ is determined by the primary speed $\omega_p$ and secondary speed $\omega_s$, i.e.

$$\omega_f = \frac{z + 1}{r_c} \omega_s - \frac{z}{r_a} \omega_p, \quad (1)$$

where $z$ is the planetary gear ratio (i.e., the ratio of annulus gear radius over sun gear radius) and where $r_c$ and $r_a$ are the carrier and annulus gear ratios, respectively. Using the definitions:

$$\tau = \frac{\omega_s}{\omega_p}, \quad (2)$$

$$\alpha = \frac{z + 1}{r_c}; \quad \tau_{GN} = \frac{z}{\alpha r_a}, \quad (3)$$

it follows that the flywheel speed can also be written as

$$\omega_f = \alpha \left(1 - \frac{\tau_{GN}}{\tau}\right) \omega_s \quad (4)$$

Obviously, the flywheel speed becomes zero whenever the CVT speed ratio $\tau$ equals $\tau_{GN}$. Therefore, $\tau_{GN}$ is called the geared neutral ratio. It is noted that at $\tau = \tau_{GN}$, the flywheel does not change the powertrain dynamics in any way, i.e., inertia effects from the primary side are neither alleviated nor exaggerated. Without proof, it is stated that another CVT ratio $\tau$ exists where the inertial torque related to speed changes at primary side is exactly compensated by the flywheel torque. For reasons apparent from [8], this ratio is termed the Zero Inertia ratio, $\tau_{ZI}$.

The torque $T_i = J_f \dot{\omega}_i$, stemming from the accelerating or decelerating flywheel (inertia $J_f$), is split by the planetary gear set into a torque $T_a$ at the annulus and a torque $T_c$ at the carrier, such that

$$T_a = \alpha \tau_{GN} J_f \dot{\omega}_i; \quad T_c = -\alpha J_f \dot{\omega}_i, \quad (5)$$

Hence, $T_a$ and $T_c$ are related by

$$T_a - \tau_{GN} T_c \quad (6)$$

From this equation the operation of the flywheel unit can be explained as follows. If $\dot{\omega}_i$ becomes negative (flywheel decelerates) then the annulus gear demands a reaction torque $T_a$ from the primary pulley given by (5) through which the carrier gear can deliver a torque $T_c$ to the secondary pulley. This torque $T_c$ is an amplification of $T_a$ if $\tau_{GN} < 1$. The inverse reasoning holds for an accelerating flywheel. The net torque $T_n$, stemming from the flywheel at the secondary pulley is given by

$$T_n = T_c + \frac{T_a}{\tau} = -\alpha \left(1 - \frac{\tau_{GN}}{\tau}\right) J_f \dot{\omega}_i, \quad (7)$$

which is obviously positive for $(\tau - \tau_{GN}) \dot{\omega}_i < 0$. Taking into account that $\dot{\omega}_i$ is typically an order of magnitude smaller than $\dot{\omega}_f$, it can be seen from equation (4) that the flywheel decelerations as the CVT ratio $\tau$ decreases, and vice versa. Decreasing the CVT ratio increases the primary speed and thus also the engine speed, resulting in primary and engine inertia $J_p$ and $J_e$ absorbing part of the torque. Provided the flywheel inertia $J_f$ is sufficiently large and the CVT speed ratio is manipulated appropriately, the engine torque $T_e$ may take the acceleration of $J_p$ and $J_e$ on its account, while the flywheel unit delivers the desired net torque $T_n$, as in (7). This is exactly the behaviour necessary to overcome the reluctance in vehicle response whenever large pedal deflections are accompanied by large leaps in engine speed. The optimal parametric design, in terms of the geared neutral ratio $\tau_{GN}$ and $J_f$ or, equivalently $\tau_{GN}$ and $\tau_{ZI}$, is treated in [8]. For the considered powertrain this resulted in $\tau_{GN} = 0.58$ and $\tau_{ZI} = 0.94$.

3 Analysis of Dynamics

To investigate the dynamics of the powertrain with and without the flywheel, it is assumed that the torque converter is locked and can be represented by a linear elastic torsion spring with a relatively high stiffness $k_d$. The finite stiffness of the drive shafts is lumped in one linear elastic torsion spring with relatively low stiffness $k_d$. Finally, the tires are modeled as a nonlinear damper, implying that traction is always accompanied by tire slip, be it from deformation or from sliding. This type of tire model is instrumental here because tire slip predominantly determines the damping of powertrain vibrations (see, e.g., [5]).

3.1 Method of analysis

The nonlinear system (7 states, 2 inputs, see [8]) is linearized around a possible stationary state. The inputs to the system are the induced engine torque $T_e$ and the CVT shift rate $\dot{\tau}$. The output considered here is the vehicle acceleration $\ddot{a}_v$.

The linear model is used to investigate some characteristics, like eigenfrequencies and eigenmodes, of the powertrain with and without flywheel. Although it is common practice to linearize around a stationary state, this is not trivial here because in a vehicle powertrain frequently large transients occur. One could also consider linearizations around an a priori given state trajectory, resulting in time-dependent system and input matrices. The choice for a stationary state is motivated by the fact that essentially the behaviour of the powertrain directly after the start of a transient is of interest here.
The linearizations are performed around several stationary states (index 0), characterized by one constant engine speed \( \omega_c \) and a number of CVT ratios \( \tau_0 \) between the lowest (UnderDrive) ratio \( \tau_{UD} = 0.42 \) and the highest (OverDrive) \( \tau_{OD} = 2.2 \). Perturbations around stationary states are indicated by tilde (\( \sim \)) The characteristic polynomial of the linearized, 7th order system has one pole \( s_1 = 0 \) in the origin, two real negative poles \( s_2 \) and \( s_3 \) and four complex poles \( s_{4,5,6,7} = \beta_1 \pm j \Omega_1 \) and \( s_{8,9} = -\beta_2 \pm j \Omega_2 \) with negative real parts \( \beta_1 \) and \( \beta_2 \). The real poles \( s_2 \) and \( s_3 \) are predominantly determined by the tire properties and the inertia of the vehicle. They hardly change if the flywheel unit is added or removed. The complex poles \( s_{4,5,6,7} \) mainly depend on the inertias of the powertrain components (including the flywheel), on the stiffness \( k_4 \) of the torque converter and the stiffness \( k_4 \) of the drive shafts. The imaginary parts (eigenfrequencies) \( \Omega_1 \) and \( \Omega_2 \) differ an order of magnitude. The lowest eigenfrequency \( \Omega_1 \) strongly depends on the stiffness \( k_4 \) and hardly on \( k_2 \), whereas \( \Omega_2 \) is determined in substance by \( k_1 \). Therefore, in the following \( \Omega_1 \) will be referred to as the drive shaft eigenfrequency or drive shaft resonance and \( \Omega_2 \) as the torque converter eigenfrequency or torque converter resonance.

3.2 The drive shaft resonance

Figure 3 shows some system characteristics at the drive shaft resonance as a function of \( \tau_0 \) for \( \omega_c = 100 \) [rad/s]. The powertrain without flywheel exhibits a strictly increasing eigenfrequency, whereas the resonance with flywheel has a maximum at \( \tau_0 = \tau_{Z1} \). The eigenfrequencies for \( \tau_0 < \tau_{Z1} \) are very similar and are exactly equal for \( \tau_0 = \tau_{GN} \). The relative damping for \( \tau_0 > \tau_{GN} \) is substantially larger for the drive line with flywheel.

The amplitudes of the transfer functions from the inputs \( T_c \) and \( \dot{a}_v \) to the vehicle acceleration deserve a closer look. The amplitude from \( T_c \) to \( \dot{a}_v \) is always smaller (or equal, for \( \tau_0 = \tau_{GN} \)) for the drive line with flywheel, the phase (not shown) is almost equal. The amplitude of the transfer function from \( \dot{a}_v \) to \( a_v \) is zero for the system with flywheel but non zero for \( \tau_0 = \tau_{Z1} \) while being larger at \( \tau_0 = \tau_{UD} \) for the system without flywheel.

3.3 The torque converter resonance

For the unmodified powertrain the torque converter eigenfrequency decreases with \( \tau_0 \), see Figure 4. This is also true for the ZI powertrain although much more pronounced from \( \tau_{GN} \) onwards. More disturbingly, the relative damping decreases rather severely for increasing \( \tau_0 \). Also, the amplitudes of the transfer functions from \( T_c \) and \( a_v \) are much larger for the ZI driveline. In light of the limited bandwidth of the CVT, this might not be too much of a problem. Furthermore, the transfer from \( T_c \) to \( \dot{a}_v \) has something of a notch around the torque converter resonance. Engine torque variations due to individual combustions (two per crank shaft revolution) are situated roughly between 30 [Hz] (for \( \omega_c = 0.100 \) [rad/s]) and 200 [Hz] (for \( \omega_c > 600 \) [rad/s]), whereas the bandwidth of the mean value engine torque varies between 1 [Hz] (for \( \omega_c = 100 \) [rad/s]) and 3 [Hz] (for \( \omega_c > 200 \) [rad/s]). Since the eigenfrequencies, as depicted in the upper left plot of Figure 4, can be close to the excitation frequency due to individual combustions, the effect of torque variations at a frequency twice the engine speed needs to be considered in more detail. Thanks to the very low relative damping of the torque converter resonance for the drive line with flywheel, the amplitude of the transfer function from \( T_c \) to \( \dot{a}_v \) evaluated at a frequency of 32 [Hz] (\( \omega_c = 100 \) [rad/s]) has fallen sharply (not shown here) to a value which is an order of magnitude smaller than the amplitude for the conventional drive line. The downside of this low relative damping is that oscillations, excited by initial conditions unequal to zero, will persist longer and will typically the same order of magnitude as oscillations induced by periodic inputs, e.g. individual combustions.

4 Analysis of Control

The ZI powertrain requires a control system to realize the ob-
jectives, being minimal fuel consumption and maximal driveability. Here, driveability is seen as a measure for the vehicle response (e.g., the acceleration pattern) directly after a kick down of the drive pedal. For the driver, only driveability is of importance in real life traffic situations. The desired behavior has to be obtained by appropriate control of the engine torque with an electronic air throttle and of the CVT ratio shift speed with a hydraulic system to manipulate the pressures on the CVT pulleys.

4.1 Control hierarchy

The total control of the ZI powertrain consists of three layers, see Figure 5.

The first layer is the driver manipulating the drive pedal to control the position, speed and acceleration of the vehicle. The second layer interprets the drive pedal deflections into setpoints for the engine torque and the CVT ratio. An important aspect of this layer is the translation of pedal deflection into desired drive torque. The philosophy behind the powertrain controller is that the driver is well able to control the position, speed and acceleration if the error between the desired and the realized drive torque is always small. The third layer involves local controllers for the engine throttle and the CVT ratio shift speed. These controllers have to take care that the setpoints, given by the second layer, are realized as good as possible. They are not addressed here in more detail. The local CVT controller is described in [12].

4.2 Controller model

The control problem now is to find setpoints for the engine torque and CVT ratio shift speed such that the fuel economy is maximized (at least in stationary situations) and driveability is maximized for all situations. A reduced order model is proposed to controller design for CVT based powertrains and for the ZI powertrain in particular. In this model it is assumed that the flexibility of the torque converter spring and the slip between the tires and the road may be neglected.

4.3 Assumptions for powertrain controller design

The goal of the powertrain controller is to realize a drive shaft torque $T_d$ that follows a time-dependent desired torque $T_{d,d}$, as close as possible, using as little fuel as possible. The controller has to cope with a number of constraints, being

- the CVT ratio is bounded by $\tau_{1UD}$ and $\tau_{1OD}$.
- the engine speed is bounded by $\omega_{e,\text{min}}$ and $\omega_{e,\text{max}}$ where $\omega_{e,\text{max}}$ is the speed at maximal engine power $P_e,\text{max}$
- for each engine speed $\omega_e$ the engine torque may not exceed the wide open throttle torque $T_{WT}(\omega_e)$.

Taking these constraints into account, the desired drive shaft torque $T_{d,d}$ is derived from the drive pedal deflection $\delta$ (scaled between 0 and 1 with $p = 1$ at maximal deflection) as:

$$\tau = \begin{cases} P(p) \cdot \text{WOT} & \text{for } \omega_s \leq \tau_{1UD} \omega_{e,\text{max}} \\ \frac{P(p)}{\tau_s \omega_s} & \text{for } \omega_s > \tau_{1UD} \omega_{e,\text{max}} \end{cases}$$

$$T_{d,d} = \kappa(\tau - T_{d,d})$$

where $T_{d,d}$ is the desired torque through the drive shaft, $\kappa$ can be seen as a 'pedal-to-wheel stiffness', $P(p)$ is a yet to be chosen, strictly increasing function of the drive pedal deflection $p$, $\tau_s$ is the final reduction gear ratio and, finally, $\omega_s$ and $\omega_e$ are the measured engine and secondary speed, respectively. Thus, dependent on whether or not the vehicle exceeds a certain minimum speed, the drive pedal deflection is translated into a desired torque via a power interpretation $P(p)$. The choice for the drive shaft torque in this translation is argued as follows. The time scale of varying vehicle loads in daily life traffic is generally large enough for the driver to take safe and appropriate actions. This fact, combined with the knowledge that common drivers are well able to control second order dynamics (e.g., see [4]), makes it reasonable to assume that the driver feels comfortable with the task of controlling the longitudinal vehicle dynamics, and less comfortable (though maybe more sportive) with powertrain dynamics. Unlike in [7], vehicle speed control therefore is not seen as a task of the powertrain controller unless explicitly demanded for by the driver by means of an enabled cruise controller. The drive shaft torque $T_d$ is exactly the quantity the driver needs to control to slow the slowly varying low order vehicle dynamics toward a desired vehicle speed or a relative distance to other vehicles in traffic. Although other solutions may apply, the translation of the pedal deflection $p$ into $T_{d,d}$ via a power interpretation as in equation (8), certifies that maximal engine power is always translated into a maximal drive shaft torque for all secondary speeds. Many authors ([9], [10], [1] and [3]) agree in translating the drive pedal deflection into a desired drive shaft torque, but few of them give an actual solution. In [11] the power interpretation is also proposed since investigation of numerous experiments showed power to have the best correlation with pedal deflection. For both cases in (8) it is assumed that longitudinal driveability is high whenever the difference between $T_d$ and $T_{d,d}$ is small for every time $t$.

4.4 Control of driveability vs. fuel economy

The powertrain controller can use the engine torque $T_e$ and the CVT ratio shift speed $\tau$ to control $T_d$. Choosing setpoints...


\[ T_{e,d} = \min(T_{PEC}(\omega_c), r_T r_{OD} T_{d,d}), \]

where \( T_{PEC} \) is the torque corresponding to the PEC. This setpoint is generally below the PEC whenever stationary torques are being delivered at \( T = T_{OD} \). The setpoint for the CVT speed ratio is governed by the ratio shifting law, as derived in [8]:

\[ r_d = \frac{(J_2 [r_d] + I_c) r_d \omega_s + r_T r_d^3 T_{d,d} - r_d^2 T_{e,d}}{(J_1 [r_d] + I_c) \omega_s}, \]

where \( J_1 = r_{GN} (r_{CN} - r) \alpha^2 J_I + J_p \), \( J_I < 0 \) for \( r > r_{ZI} \), and \( J_2 = (r_{CN} - r)^2 \alpha^2 J_I + J_p + r^2 J_s \) (\( J_s \) is the secondary inertia), and where \( \omega_s \) is a reconstruction of the secondary pulley acceleration.

For the Z1 powertrain, \( J_1 [r] + I_c \) is negative for ratios between \( r_{OD} \) and \( r_{ZI} \). Within this range, the control law (10) generates a shifting towards lower ratios for torque demands higher than can actually be delivered by the engine. This downshifting brings the engine power towards a higher level by increasing the engine speed and, more importantly, during the shifting itself it exactly fills up the engine torque deficiency. For ratios \( r_d \) approaching \( r_{ZI} \), control law (10) can not be used because \( J_1 [r_d] + I_c \) then approaches zero. Controlling the CVT ratio for the Z1 powertrain then raises the same type of difficulties as for the unmodified powertrain. For unmodified CVT powertrains \( J_1 [r] + I_c \) is always positive and control law (10) would result for all possible CVT ratios in an unwanted upshift for torque demands higher than actually can be delivered by the engine. In practice, well behaving controllers are developed that both induce a downshift and relax the ratio shift speed to assure a strictly increasing though less time-optimal torque response in the drive shaft. Temporary deviations from the PEC up to the WOT line will increase the driveability although it is typically traded for fuel economy. The solution chosen here is to saturate \( J_1 [r_d] + I_c \) in (10) at some value smaller than zero. This will indeed relax the shift speed. Within the operation envelope of engine and vehicle, driveability results are fair. Note that the equivalent primary inertia \( J_1 [r] + I_c \) will become positive for \( r \) between \( r_{ZI} \) and \( r_{CN} \) although it is still less than for a conventional CVT powertrain. Ratios below \( r_{CN} \) only occur during vehicle launch and during fast accelerations from low vehicle speeds. As discussed in [8], the performance penalty for this case is minimized in the current design of the flywheel unit and is, anyhow, for the larger part determined by the primary inertia.

**4.5 Simulation results**

In this section simulation results are shown, using three different strategies for determining the setpoint for the engine torque. The simulations are performed using the nonlinear model referred to in Section 3, augmented with non-linear second order CVT dynamics and slightly less than critically damped second order mean value engine torque dynamics. Some of the result, stemming from an arbitrary excursion of pedal deflection, are presented in Figure 7 and Figure 8.

Three engine torque strategies are simulated. The first strategy, indicated by ‘a’, is given by equation (9) and tracks the PEC also in transients. Strategy ‘b’ can deviate from the PEC to a torque level immediately corresponding to the aimed stationary value on PEC. The third strategy, indicated by ‘c’, controls the engine torque along the WOT line as long as \( T_{d,d} \) is not reached yet. Strategies similar to b and c, also referred to as “Off the Beaten Track”, were proposed earlier in [11] for the so-called economy and sport mode for CVT-equipped passenger vehicles.

Excursions of \( T_e \) using these three strategies can be seen in the lower right plot in Figure 7. The upper left plot shows the desired and realized power for the three strategies. The upper right plot shows a fragment thereof and the lower right plot gives the same fragment but now displayed in the engine map. The cumulative fuel consumption for the three strategies is presented in the lower left plot, whereas the engine speed, flywheel speed and vehicle speed as well as acceleration can be seen in Figure 8. From these plots the following observations and conclusions evolve:

- The driveability with flywheel is about the same for all three strategies. The differences in engine torque are compensated by the ratio shifting law (10), whereas the fuel consumption is slightly higher (ca. 0.9 %) for strategy c.
- Clearly, the driveability without flywheel hesitates and more promptly displays the drive shaft resonance.
- The torque converter resonance with flywheel seem acceptably low in magnitude.
- The vehicle position advantage with flywheel amounts up
to 9 meters due to the difference in vehicle speed history (upper right, Figure 8).

- As explained in paragraph 4.4, full inertial torque compensation is possible only for $T > T_{Z1}$. For $T < T_{Z1}$, strategy c might alleviate a slight reluctance in performance, with a small fuel economy penalty.
- The initial operating point of the engine is not at all on PEC. This is caused by the CVT ratio constraint $T \leq T_{OD}$.
- The lower right plot in Figure 8 shows that the response in vehicle accelerations is fairly smooth though immediate.
- To obtain these smooth results the reconstruction of $u$ used in equation (10) was filtered using a $1 \text{ [rad/s]}$ bandwidth first order filter. This bandwidth is well below the drive shaft resonance, ensuring this resonance is not amplified by the CVT actuation.

5 Conclusions

A system description of a new CVT powertrain with an extra flywheel is presented. This flywheel fills the driveability gap of conventional CVT powertrains that are controlled for maximal fuel economy. Dynamic analysis showed that the leading resonance modes of the new powertrain are quite different. Oscillations induced by the torque converter may be more apparent though still small in magnitude. Oscillations induced by the drive shafts receive extra (virtual) damping up to a factor three by the additional flywheel. A powertrain controller was derived, generating setpoints for engine and CVT from the course of the drive pedal deflection. This controller succeeds in controlling the engine speed at the lowest possible values for stationary situations while delivering smooth though immediate acceleration responses of the vehicle. The system has already been tested extensively and successfully on a test rig. In the near future, the objective and subjective response of a real vehicle with the new powertrain will be evaluated.

Acknowledgments

This study is part of the EcoDrive project, a joint project of Van Doorne’s Transmissie (VDT), Netherlands Organization for Applied Scientific Research (TNO) and the Technical University Eindhoven (TUE). The project is subsidized by the Dutch governmental program E.E.T. (Economy, Ecology and Technology).

References