Methods

The rate of change of bone volume fraction in a RVE (Figure 2), modulated by mechanobiological and geometric feedback, can be expressed by:

$$\frac{d(BF)}{dt} = \left[ \left( \mu \cdot \frac{MS}{BF^2} \right) - f_{resocl} \cdot V_{f} \right] \alpha \frac{2}{R} \cdot BF$$

Eq.(1)

where $\tau$ is the bone formation time constant [mm$^3$/nmol/s/day], $\mu$ is the osteocyte mechanosensitivity [(nmol/s)/(MPa/s/mm$^2$)], $\alpha$=0.8 and $\gamma$=3 are coefficients, MS is the strain energy density rate [MPa/s], $f_{resocl}$ is the osteoclast recruitment frequency [1/day] and $V_{res}$ is the linear resorption per cavity [mm]. A previously developed bone remodeling algorithm based on mechanotransduction [1,2] was used to compare analytical and numerical results.

Results

The closed-form solution of Eq. (1) is represented by:

$$BF(t) = \left\{ \frac{MS \cdot \mu \cdot \tau + c \cdot f_{resocl} \cdot V_{f} \cdot MS \cdot \mu \cdot \tau}{(f_{resocl} \cdot V_{f})^2} \right\}^{\frac{1}{\alpha}}$$

The evolution of bone volume fraction is plotted in Figure 3A,B for two different initial conditions. Theoretical predictions and numerical results show close matching. It follows that the presented analytical model is suitable for guiding the computer simulations of bone microstructures in different loading and metabolic conditions (Figure 3C,D).

Discussion

In this study the bone remodeling was modeled as a dynamic process which was driven by mechanical loading and cellular activities. The analytical model allows for investigating the interaction between metabolic and mechanical factors and for studying the effects of nutritional, hormonal and pharmacological therapies in different diseases.