Observer design for a nonlinear two-dimensional pool boiling system

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1 Introduction
In pool-boiling systems heat is extracted from a heater by a pool of boiling liquid. The heat flux between heater and fluid is highly nonlinear with respect to heater temperature, described by \( q_F(T_F) \) (Figure 1) and results in a highly unstable regime which must be stabilised to allow for high heat removal rates. This can be accomplished by a control law based on the (nonmeasurable) spectral modes of the heaters temperature field \([1]\). Application of this control law requires an observer for the temperature profile of the heater.

**Figure 1 Nonlinear heat flux \( q_F \) as function of interface temperature \( T_F \).**

2 Pool boiling model description
The heat transfer in the 2D rectangular nondimensional heater \( \mathcal{H} := \{(x,y) \in [0,1] \times [0,D]\} \), see Figure 2, is considered. Its temperature field \( T(x,y,t) \) is described by

\[
\frac{\partial T}{\partial t}(x,y,t) = \kappa \nabla^2 T(x,y,t). \tag{1}
\]

The boundary conditions comprise adiabatic, i.e. perfectly isolated, sidewalls on \( \Gamma_A := \{(x,y)|x = 0,1\} \), a controlled heat supply on \( \Gamma_H := \{(x,y)|y = 0\} \) and the nonlinear heat extraction on \( \Gamma_F := \{(x,y)|y = D\} \), i.e.

\[
\frac{\partial T}{\partial x}\big|_{\Gamma_A} = 0, \quad \frac{\partial T}{\partial y}\big|_{\Gamma_H} = -1 + u(t), \quad \frac{\partial T}{\partial y}\big|_{\Gamma_F} = -\Pi_2 q_F(T_F) \quad \tag{2}
\]

Here \( T_F(x) := T(x,D) \) is the fluid-heater interface temperature, \( \Lambda, D, \Pi_2 \) and \( \kappa \) are positive parameters, \( q_F(T_F) \) is the boiling curve and \( u(t) \) is the input, i.e. an additional heat supply at the bottom of the heater, see \([1]\).

**Figure 2 Two-dimensional rectangular heater.**

3 Observer design
The temperature can be measured at \( R + 1 \) points on the heater surface, given by \( \tilde{x}_r = r/R \), \( r = 0,\ldots,R \), i.e. the system output equals \( y_r = T_F(\tilde{x}_r) \) for \( r = 0,\ldots,R \). The observer is designed to be a copy of the system with output injection only on the boundary condition where the measurements are available. The obtained observer is given by (1) and (2) with the state \( Z(x,y,t) \), i.e. the estimate of \( T(x,y,t) \), instead of \( T(x,y,t) \) and with the boundary condition on \( \Gamma_F \) as

\[
\frac{\partial Z}{\partial y}\big|_{\Gamma_F} = -\Pi_2 q_F(Z_F) \quad \tag{3}
\]

with \( p_r(x) \) the observer gain functions. The observer error dynamics of \( E = T - Z \) are analysed by linearisation of system and observer, with which local stability of the nonlinear error dynamics can be obtained. Figure 3 shows the evolution of the output, for \( R = 2 \), meaning the output equals \( T(0,D,t), T(0.5,D,t) \) and \( T(1,D,t) \). The initial system state equals a uniform field of \( T \approx 4 \), whereas the initial observer state equals zero. Although the initial differences between system state, observer state and non-uniform equilibrium are large, the observer states converge to the system states and the equilibrium is stabilised.

**Figure 3 Evolution of the system output and the observer output.**

4 Conclusion
An observer for a nonlinear PDE system is designed. The closed-loop system is stabilised by the control law discussed in \([1]\) and this observer, even for large initial perturbations.

References