Finite Element Modeling of an Ultrasonic Transducer

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Literature Survey

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Abstract

Ultrasonic transducers are integrated into the minimally invasive needles and catheters for medical diagnostic. There is a need for a model that can predict the behavior of the ultrasonic transducer. Many designers employ one dimensional analytic models, however due to certain limitations these models are not adequate to understand the entire performance of an ultrasonic transducer.

Nowadays, finite element modeling is being adopted in the design of ultrasonic transducers. The purpose is to get more realistic transducer simulations and to accelerate the design process. In this report, a literature survey is presented in order to model the complete ultrasonic measurement system using the finite element method.
Preface

This literature survey is written as a part of the graduation thesis in Dynamics and Control group at department of Mechanical Engineering, Eindhoven University of Technology (TU/e). It is pleasure for me to thank all those who made this possible. First of all, I owe my deepest gratitude to Prof. Henk Nijmeijer (TU/e) who made his support available in a number of ways and made it possible to work on such an interesting topic in Philips Research (High Tech Campus).

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Chapter 1

Introduction

In this chapter, we discuss an application for which we need to model the ultrasonic transducer in an ultrasonic measurement system. This provides motivation to develop a basic understanding for the ultrasonic measurement system.

1.1 Cardiac Ablation

The heart is a pump that functions by pumping the blood in a controlled sequence through its four chambers: left atrium, right atrium, left ventricle and right ventricle. Each heart beat originates as an electrical impulse from a small area of tissue in the right atrium of the heart called the sinus node. The impulse initially causes both of the atria to contract, then activates the atrioventricular node which is normally the only electrical connection between the atria and the ventricles. The impulse then spreads through both ventricles causing a synchronized contraction of the heart muscle.

For a healthy person, this regular flow of electricity through defined paths is the basis of normal heart muscle contractions which results in normal heart beat. However, sometimes this flow of electricity becomes irregular due to a faulty electrical pathway and, consequently, the heart beats very erratically. Such heart diseases are termed arrhythmia in medical science. In adults the normal resting heart rate ranges from 60 to 80 beats per minute. The resting heart rate in children is much faster.

Medicine often helps in various cases for the treatment of arrhythmia. However, an effective treatment is to destroy a heart tissue in a controlled manner. Consequently, sources of unwanted pulses or electrical pathways are destroyed. Cardiac ablation is a minimally invasive procedure in which a catheter—a thin flexible tube as shown in figure 1.1— is inserted in the blood vessel and then moved into the
heart by surgeons. Once the tip of the catheter has reached the targeted place into
the heart, Radio Frequency(RF) energy is used to destroy the tissue.

![Figure 1.1: A typical catheter used for ablation [Courtesy: Philips Research]](image)

During cardiac ablation, there is a potential risk to destroy the complete heart wall
due to extra energy provided by an ablation electrode. There is a need to monitor
the wall thickness and to measure thickness of lesion– abnormal tissue– throughout
the ablation process as shown in figure 1.2. In Philips Research, the group of
Minimally Invasive Healthcare integrates the ultrasonic transducer in needles and
catheters so that lesion monitoring and ablation can be done simultaneously.

![Figure 1.2: Cardiac ablation process [Courtesy: Philips Research]](image)

An integrated ultrasonic transducer sends acoustic waves into the heart wall and
receives an echo to characterize the heart tissue. The signal received back from the
transducer is processed and an image is created.

### 1.2 Ultrasonic Measurement System—System Description

An ultrasonic measurement process involves the generation of ultrasound by the
transducer, propagation of these ultrasonic waves into the propagating medium
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and then reception of these waves through the transducer again. The complete measurement system consists of a pulser, transducer and receiver. The pulser sends the electrical pulse via a cable to an ultrasonic transducer. The transducer converts the electrical pulse into an acoustical pulse in the generation system and an acoustical into an electrical signal in the reception system. In addition, a 50 Ω attenuator can be used in between the transducer and the pulser/receiver in order to avoid the electrical reflections.

When the transmission and reception of the acoustic waves occur via the same ultrasonic transducer, the method is called pulse echo mode as shown in figure 2.12. This method is used for cardiac imaging through a catheter.

1.3 Report Outlook and Organization

This chapter discusses an application for which we need to model an ultrasonic transducer. It then describes the basic ultrasonic measurement system in order to understand its working principle.

Chapter 2 reviews the literature which is required to model an ultrasonic transducer using the commercial software package COMSOL. The chapter elaborates various issues that affect simulation strategies. The dependence of the simulation results on accurate model parameters including material properties, adequate mesh resolution, realistic boundary conditions and appropriate geometrical simplifications are discussed. In addition, a detailed experimental validation procedure along with the performance measurement techniques is elaborated. Chapter
Introduction

3 summarizes the salient features of the complete discussion and gives concluding remarks.
Chapter 2

Literature Survey

2.1 Transducer Design

Traditionally, the design of an ultrasonic transducer is normally accomplished via established "rules of thumb" and fundamental theoretical understanding. After an initial design is made, the performance of the transducer is analyzed either by a simple analytic one-dimensional model or a three-dimensional finite element model. Before we start with the design guideline for a single element transducer, it is necessary to understand the basic configuration of the transducer.

2.1.1 Basic configuration of the ultrasonic transducer

For cardiac applications, the outer diameter of the catheter is between 7F and 9F (3F = 1mm), and an ultrasonic transducer should fit in such catheter. An ultrasonic transducer represents a layered structure as shown in figure 2.1. The basic component of a transducer is piezo crystal which converts the electrical energy into mechanical (acoustic) energy and vice versa. In order to support the piezo crystal, a backing plate is used at the back of the piezo crystal. One or more front layers can be used to improve the power transmission between the piezo crystal and the propagating fluid. These layers also acts as wear protection plates for a piezo element. The diameter of the piezo crystal is 1 mm. A simplified construction of an ultrasonic transducer is shown in figure 2.1.

2.1.2 Quarter-wave acoustic matching

Usually for short impulse response and broad bandwidth, ultrasonic transducers are designed with one or several matching layers at the front [1]. These quarter-
wave front layers are used to improve the energy transmission from the piezo crystal to the acoustic medium. For designing an ultrasonic transducer for in-vivo (within the living) applications, material of the front layer should also be bio-compatible.

McKeighen in [1] mentions that the transmission line theory can be used to choose the front layers such that these layers provide matching between impedance of the piezo ceramic $Z_{\text{piezo}}$ and impedance of the acoustic medium $Z_{\text{med}}$. According to transmission line theory, the optimal acoustic impedances for one or two matching layer are:

For one layer:

$$Z_{\text{layer}} = \sqrt{Z_{\text{piezo}}Z_{\text{med}}}$$  \hspace{1cm} (2.1)

For two layers:

$$Z_{\text{layer1}} = Z_{\text{piezo}}^{3/4}Z_{\text{med}}^{1/4}$$
$$Z_{\text{layer2}} = Z_{\text{piezo}}^{1/4}Z_{\text{med}}^{3/4}$$  \hspace{1cm} (2.2)

For the design of a wide-band transducer, Desilets et al. in [2] modify the choice of front layers using the KLM theory. They consider the first half of a piezoceramic as a quarter-wave matching layer, in addition to the quarter wave matching layers attached to the ceramic. Their analysis dictates the choice of the matching layers as:

For one layer:

$$Z_{\text{layer}} = Z_{\text{piezo}}^{1/3}Z_{\text{med}}^{2/3}$$  \hspace{1cm} (2.3)
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For two layers:

\[
\begin{align*}
Z_{layer1} &= Z_{piezo}^{4/7} Z_{med}^{3/7} \\
Z_{layer2} &= Z_{piezo}^{1/7} Z_{med}^{6/7}
\end{align*}
\] (2.4)

The addition of more than one matching layer increases the complexity to manufacture the transducer. However, this also increases the efficiency of the transducer and results in the improvement of both the bandwidth and sensitivity.

2.1.3 Backing layer

To select an appropriate backing layer, there are several design considerations which need to be considered. Brown in [3] discusses in detail about these design considerations. Based on that discussion, the following design considerations are necessary to take into account.

1. The impedance of the backing material has to be according to the required bandwidth of the ultrasonic transducer. For the cardiac application, we need high bandwidth transducers. This gives a fast ringing down of the signal and, a higher resolution for imaging. However, increasing the bandwidth with an increase of the backing impedance, also decreases the efficiency [4] and thus the signal to noise ratio (SNR). This means there is a tradeoff between these parameters.

2. The attenuation coefficient of the backing material should be as high as possible so that acoustic waves transmitted to the back cannot reflect back and we receive an echo without interference.

There are other design considerations which do not influence the performance of the ultrasonic transducer but these are necessary for manufacturing the transducer.

1. The backing material should be easy to machine and should be shaped into different thickness and size.

2. The backing material should have good adhesion properties so that it can adhere with the piezoelectric material.

3. The backing material should have high surface quality to ease the transfer of energy to the back and to reduce noise.
2.1.4 Electrical matching

The electrical impedance of an ultrasonic transducer depends on the material properties of the active element (piezo ceramic) as well as the effects of the front matching layers and the backing layer [5]. There is a high chance that the impedance of a designed transducer does not match with the impedance of the connected cable and transceiver (pulser/receiver) at the transducer resonant frequency. This electrical impedance mismatch creates the electrical reflections between the transducer and cable and between the cable and transceiver. In order to avoid these electrical reflections, which can cause artifacts in the image, the following solutions are suggested in [5].

1. The basic electrical elements (preferably capacitors and inductors) can be placed in series or shunt (as per the design requirement) between the cable and transducer and/or between the cable and transceiver to match the electrical impedances.

2. A coaxial cable itself can be used to match the impedance of the transducer with the transceiver. In this approach, the electrical impedance of the transceiver \( Z_{elec} \) and the electrical impedance of the transducer \( Z_{tran} \) at the transducer resonant frequency dictate the choice of the electrical impedance of the cable as given below.

\[
Z_{cable} = \sqrt{Z_{elec}Z_{tran}}
\]  

(2.5)

In order to transfer the maximum power, the length of the cable should be equal to the quarter wavelength of the signal within the cable.

Any of the above "rules" not only removes the electrical reflections but also increases the energy transmission, which consequently improves the signal to noise ratio and bandwidth of the received echo signal.

2.2 Transducer Modeling

2.2.1 One-dimensional modeling

Two well known one-dimensional analytic models are Mason’s and the KLM model, see [6]. These models are equivalent circuit models for a piezo crystal and use different electrical configuration, see [7]. The transducer has a backing layer at the back and single (or multiple) layer(s) at the front side. In order to model the complete transducer, we also need a model of the backing layer and the front layer
separately. For the detailed modeling process and implementation using the Mason’s model, see [8]. Both Mason’s and KLM model are one-dimensional in nature and are very effective in predicting the response of an ultrasonic transducer. These models are computationally very cheap and help to accelerate the product design cycles. Besides many advantages, these models have the following limitations as well:

- The model is one-dimensional, therefore only a one-dimensional pressure field can be investigated.
- The model assumed that the thickness of the backing layer is semi-infinite without losses [9]. This means that any acoustic wave transmitted to the back does not reflect back from other interfaces. In reality, these reflections can interfere with the received echo from the front side, if the backing layer has less attenuation.
- The model is made for a piezo crystal having the lateral dimensions much larger than the thickness. In practice, due to severe space limitations, the piezo crystal does not satisfy these geometrical constraints. Consequently, the model results will become unreliable and less accurate.
- The model is made for a thin, loss-less, disc shaped piezo crystal. It is not valid for a lossy piezo ceramic and polymer based piezo element. For this reason, several authors (such as [10] and [11]) modified the actual electrical configuration of the standard circuit to introduce losses inside a piezo crystal.
- The model is made to simulate an ultrasonic transducer. It cannot simulate the interactions of other objects with the transducer (such as catheter cap shown in figure 2.1).

2.2.2 Three-dimensional modeling

A finite element model for an ultrasonic transducer provides a realistic transducer simulation and gives a way to visualize the real acoustic wave propagation into the acoustic medium [12]. Using the finite element method, we can cope with all the limitations described above for one-dimensional model, such as:

- Using the finite element model a three-dimensional pressure field can be investigated.
- Finite element models are based on the governing equations of the acoustic propagation and piezoelectricity, so back reflections from any interface can be modeled and investigated.
• With the 3D or 2D axisymmetric models, we can simulate all the vibration modes upon the excitation of a piezo crystal layer. There is no geometrical constraint for accurate simulations.

• The losses in the materials (both in active and passive materials) can be modeled easily by setting the appropriate material properties.

• The interaction of the acoustic field of an ultrasonic transducer with other objects can be simulated.

Besides the above advantages, finite element modeling has certain disadvantages as well. In order to obtain required accuracy at high frequencies, the finite element model of an ultrasonic transducer comprises of several thousands to few million degrees of freedom. Such a large model requires computer systems with extensive processing capabilities and considerable effort to build, debug and operate.

Another disadvantage of finite element modeling is that the response of a transducer can differ substantially from the actual response. This can be due to the use of an inadequate number of elements to resolve acoustic waves, unrealistic boundary conditions or less accurate model parameters (such as material properties). These disadvantages indicate the need for experimental validation of a finite element model. This will be discussed in a latter section.

2.3 Vibration Characteristics of a Piezoelectric Disc

It is a common practice to analyze the vibration characteristics of the piezoelectric disk through one-dimensional analytic models. These one-dimensional models assume that the piezoelectric disk vibrates in thickness extensional mode and behaves like the motion of a piston as shown in figure 2.2.

![Figure 2.2: The thickness extensional mode shape of a piezo disc assumed by the one-dimensional models. Solid line- undeformed disc, dashed line- deformed disc, Ref. [13]](image)
Guo et al. mention in [13] that these one-dimensional methods are applicable to a piezo disk with diameter to thickness \((d/h)\) ratios greater than or around 20. This is due to the fact that under such geometrical constraint, the vibration characteristic related to the thickness extensional mode becomes dominant. However, many designers employ the disc with the \(d/h\) ratios much less than 20 due to space limitations in their design. In these cases, the use of the finite element method to analyze the vibration characteristics of a piezoelectric disc becomes indispensable.

Guo et al. in [13] present typical mode shapes for a piezoelectric disc with \(d/h\) ratio of 20, and identified five types of modes based on their characteristics. These modes are radial (R), edge (E), thickness shear (TS), thickness extensional (TE), and high frequency radial (A) modes as shown in figure 2.3. Due to axisymmetry, only half of the cross-section is shown with the axial symmetric axis on the left. The details for each of these modes can be found in [13]. Furthermore, it is shown that none of the found modes behaves like a pure piston-like motion, as assumed in the one-dimensional theory.

![Figure 2.3: Five types of mode shapes identified by Finite element analysis for a piezo disc with \(d/h=20\).](image-url)

(a) Radial mode, (b) Edge mode, (c) Thickness shear mode, (d) Thickness extensional mode and (e) High frequency redial mode, Ref. [13]
It implies that the one-dimensional models are no longer adequate to completely analyze the performance of an ultrasonic transducer. It is therefore necessary to analyze the piezoelectric transducer using a complete three dimensional model. The finite element method (FEM) has a great flexibility to analyze the piezo element disc independently and with the layered structured inside an ultrasonic transducer.

2.4 Finite Element Modeling of the Piezoelectric Ultrasonic Transducer

An ultrasonic measurement process involves the generation of ultrasound by the transducer and propagation of the acoustic waves into the propagating medium. It is important to see the formulation of the governing equations for the piezoelectric device and wave propagation in the acoustic medium.

2.4.1 Formulation of the piezoelectric effect

2.4.1.1 The piezoelectric effect

The piezoelectric effect involves the conversion of electric energy to mechanical energy and vice versa. The direct and reverse piezoelectric effects are shown in figure 2.4. The direct effect consists of the generation of net charge across the electrodes upon the application of stress, whereas the reverse piezoelectric effect refers to the phenomenon in which an applied electric potential across the electrodes induces a deformation of the crystal.

2.4.1.2 The piezoelectric constitutive relations

Abboud et al. in [15] express the governing equations of the linear piezoelectricity along with the equations of the mechanical and electric balance as:

Constitutive equations:

\[ T = c E.S - e^T.ED = e.S + \mathcal{E}_S.E \]  \hspace{1cm} (2.6)

Momentum balance:

\[ \rho \ddot{u} = \nabla . T \]  \hspace{1cm} (2.7)

Electric balance:

\[ \nabla . D = 0 \]  \hspace{1cm} (2.8)
With $S = \nabla^s \cdot u$ and $E = - \nabla \phi$

Here, $T, S, E, D$ are the mechanical stress, mechanical strain, electric field and electric displacement, respectively. $c_E, \varepsilon_S, e$ are the matrices of stiffness constants at constant electric field, dielectric constants at constant strain, and piezoelectric coupling constants, respectively. Furthermore, $u$ is the mechanical displacement vector and $\phi$ is the electric potential. With the appropriate boundary conditions, such as the specified displacement or potential, the problem can be formulated as required.

### 2.4.2 Formulation of the acoustic waves propagation

In [16], the basic mathematical formulation for acoustic waves propagation is described, which is summarized here in view of the current modeling requirements. Acoustic waves in a ‘lossless’ medium are governed by the following equation:

$$
\frac{1}{\rho_o c^2} \frac{\partial^2 p}{\partial t^2} + \nabla \cdot \left( -\frac{1}{\rho_o} (\nabla \phi - q) \right) = Q
$$

(2.9)
Here, $p$ is the acoustic pressure in the medium, $\rho_0$ is the density of the acoustic medium, $c$ is the speed of sound in the medium, $q$ and $Q$ are the monopole and dipole sources respectively.

The acoustic medium with lossy behavior can be modeled by using a complex speed of sound and a complex density. Various fluid models are available in the COMSOL (a commercial FEM package) such as linear elastic, linear elastic with attenuation, Delany-Bazley (for a porous medium), viscous medium and ideal gas models. Each of these models has its own formulation, details can be found in [16]. When there is a need to model the transient behavior, only certain frequency dependencies can be modeled, which limits the number of fluid models. In time domain an additional term of first order time derivative can account for attenuation of the sound waves, as shown below:

$$
\frac{1}{\rho_0 c^2} \frac{\partial^2 p}{\partial t^2} - d_a \frac{\partial p}{\partial t} + \nabla \cdot \left( -\frac{1}{\rho_0} \left( \nabla p - q \right) \right) = Q 
$$

(2.10)

Here, $d_a$ is the damping coefficient

The quality and accuracy of the results is dependent on the accurate boundary conditions. Below are typical boundary conditions which can be used during an modeling exercise,

- Sound hard boundary - Zero acceleration condition
- Sound soft boundary - Zero acoustic pressure condition
- Acceleration boundary condition - Specified normal acceleration at the boundary
- Pressure boundary condition - Specified acoustic pressure at the boundary
- Continuity boundary condition - Equal acceleration at the interface between parts in an assembly
- Impedance boundary condition - Specified impedance condition at the the boundary
- Non reflecting boundary condition - To allow an outgoing wave to leave the medium with minimum reflections
- Axial symmetry boundary condition - For axisymmetric geometries to reduce the dimensions
2.4.3 Finite element method

We have seen that governing equations for both the piezoelectric effect and acoustic wave propagation involve partial differential equations. The finite element method is one of the possible numerical techniques that can be used to find the approximate solutions for partial differential equations. Nowadays, thanks to the commercially available packages (such as COMSOL, ANSYS, ABAQUS, PZFLEX etc.) for solving complex problems by the finite element method. However, a modeler or designer needs to know various aspects involve in the finite element modeling to accurately model and analyze a problem. Many authors including [15], [17], [12], [18] etc. adopted finite element method to analyze the ultrasonic transducer.

Modeling of an ultrasonic measurement system using a finite element approach involves various issues. This includes a careful selection of the mesh resolution to resolve waves, artificial boundary conditions and model connections to the real world (surroundings), damping of the the real world materials etc. In what follows, we discuss some of the fundamental modeling issues underlying the finite element simulation of the ultrasonic transducer propagating in the acoustic medium.

2.4.3.1 Spatial discretization

The resolution of a solution provided by the finite element model is defined by its mesh. The solutions of the wave propagation problems are wave like in nature. These waves are characterized by the wavelength $\lambda$ in space. The dependence of wavelength on the frequency $f$ and speed of sound $c$ in a acoustical medium is given by:

$$\lambda = \frac{c}{f}$$ (2.11)

For wave propagation problems, the spatial discretization must be defined such that it can resolve the shortest wavelength (i.e. highest frequency) of interest [15]. To have a meaningful solution on the discrete grid, at least two degrees of freedom per wavelength is required [16]. However, such a coarse solution is useless to analyze. In practice, the designer needs adequate resolution of the solution. In [16], it is recommended to use 12 degrees of freedom per wavelength to get an adequate solution rather than a borderline solution. As in most cases, the direction of propagation is not known before analysis, so it is better to use the isotropic mesh. This gives the following rules of thumb to calculate the number of degrees of freedom (DOFs).

- For 1D, Number of DOFs = 12 times the length of a model geometry measured in wavelengths
• For 2D, Number of DOFs = 144 times the area of a model geometry measured in wavelengths squared

• For 3D, Number of DOFs = 1728 times the volume of a model geometry measured in wavelengths cubed

Nowadays, a 32-bit computer system can usually deal somewhere from a few hundred thousand up to a million degrees of freedom [16], using the commercial software package COMSOL. It is a good practice to estimate the number of degrees of freedom in a model before hand, as it can help the modeler to take certain decisions regarding the model simplifications.

2.4.3.2 Analyzing model convergence

Convergence of a finite element model is necessary to obtain an accurate solution. Although the rules of thumb indicated above (i.e. use 12 DOFs per wavelength) is adequate, it is recommended in [16] to perform a convergence test to check if the mesh density is sufficient. In such a test, a modeler refines the mesh, runs the study again, and then checks if the solution converges to a stable value. If the solution changes after the mesh refinement then it means that the solution is mesh dependent. Such an unstable solution dictates the need of a finer mesh. This convergence test should be performed until the solution converges to a stable value. The commercial FEM package COMSOL offers adaptive mesh refinement, which adds mesh elements based on an error criterion to resolve those areas where the error is large.

2.4.3.3 Axial symmetry

When the geometry of the ultrasonic transducer has cylindrical shape, it is impractical to model the full 3D model of the transduction device and surrounding acoustic media for the axial symmetric geometries. Usually, the transducer in a catheter has a circular piezo ceramic disc. So, we can take advantage of the axial symmetry and model the transducer in a 2D domain. In figure 2.5, a 2D axisymmetric model of a transducer enclosed in a catheter is shown. By using a 2D axisymmetric geometry instead of the complete 3D geometry, we can reduce the number of degrees of freedom dramatically and save a lot of computational resources. Many authors including [12] have taken advantage of axisymmetric geometry of the transducer and modeled the transducer with a 2D axisymmetric geometry. Furthermore, Guo et al. in [13] found the vibration characteristics of a piezo disc through a 2D axisymmetric geometry.
2.4.3.4 Artificial boundary conditions

In most cases, it is required to simulate the acoustic wave pattern of a transducer which is not contained in a closed cavity. However in practice, the acoustical domain has to be restricted to the area of interest in the model. There is a need to separate the model from its surrounding (everything outside the model). It can be achieved by imposing the truncation boundary conditions which can simulate the behavior of a continuing medium [15]. This means that the wave energy must propagate out of the computational domain without any spurious reflections, consistent with the fact that there is no impedance mismatch at the boundary. Boundaries of this kind are generically referred to as ‘Artificial Boundary Conditions’ [16]. In figure 2.6, a representation of an artificial boundary condition can be seen. In COMSOL, these kind of boundary conditions are termed ‘Non-reflecting boundary conditions’. A drawback related to these boundary conditions is that these are not perfectly non-reflecting [16] and can introduce spurious reflection. Besides the computational advantage of spending less number of degrees of freedom, a careful and appropriate use of the non-reflecting boundary conditions is demanded for modeling.

2.4.3.5 Evaluation of the acoustic field in the far field region

It is often needed to evaluate the acoustic field at a distance far away from the transducer. In this case, a solution by the finite elements would require the field
calculations everywhere in the connecting region between the source and the point of interest [15]. This is computationally very expensive and consumes a lot of time to solve the problem, which is often impractical and not necessary. An alternative approach, that often use in practice, to solve the acoustic field is the use of an exterior integral formulation. For a homogeneous medium, the solution anywhere outside a closed surface containing all the sources can be written as a boundary integral in terms of pressure and its normal derivatives (which is related to the velocity) on the surface [16]. The boundary between the near field and far field is not a sharp edge, but for engineering purpose the following definition can be used to define the far field [16].

\[ R > \frac{8a^2}{\lambda} = \frac{8ka^2}{2\pi} \]  

Here, \( a \) is the radius of sphere enclosing all the sources and objects, \( \lambda \) is the wavelength and \( k \) is the wave number. In what follows, we discuss the mathematical formulation to evaluate the results in the far-field based on [16].

**The Helmholtz-Kirchhoff Integral Representation-Mathematical Formulation**

The acoustic Helmholtz’s equation is given by:

\[-\nabla \cdot \nabla p - k^2 p = 0\]  

In the homogeneous domain exterior to a closed surface \( \mathbf{S} \), the solution \( p \) to the above equation can be explicitly expressed as:
\[ p(R) = \int_S \left( G(R, r) \nabla p(r) - \nabla G(R, r) \cdot p(r) \right) \cdot n dS \tag{2.14} \]

Here, the unit vector \( n \) is directed into the domain enclosed by \( S \), which means \( n \) points normal outwards to the exterior infinite domain. The close surface \( S \) is parameterized by the coordinate vector \( r \). The function \( G(R, r) \) is the Green’s function satisfying the following equation:

\[-\nabla \cdot \nabla G(R, r) - k^2 G(R, r) = \delta^{(3)}(R - r) \tag{2.15}\]

The Green’s function itself is a function of the vector \( r \) of an outgoing wave excited by a source at \( R \). For the 3D case, this Green’s function can be written as:

\[ G(R, r) = \frac{e^{-ik|r-R|}}{4\pi|r-R|} \tag{2.16}\]

Substituting the above Green’s function into the general solution for the pressure (equation (2.14)), we have the following expression for the solution:

\[ p(R) = \frac{1}{4\pi} \int_S \frac{e^{-ik|r-R|}}{|r-R|} \left( \nabla p(r) + p(r) \frac{1 + ik|R-r|}{|r-R|^2} (r-R) \right) \cdot n dS \tag{2.17}\]

For the 2D axial symmetric geometry, evaluation of the 3D integral is necessary which needs 3D geometry. To accommodate this problem, COMSOL uses an adaptive numerical quadrature in the azimuthal direction on a fictitious revolved geometry, in addition to the standard mesh-based quadrature in the 2D rz-plane [16].

### 2.4.4 The Electric Circuit

In an pulse-echo ultrasonic measurement system, a transducer is attached to a transceiver (pulser and receiver) usually through a coaxial cable. It is necessary to model the cable and transceiver circuit. Abboud et al. in [15] have shown the generic circuit for a commercial pulser and receiver, and describe that this generic circuit can be modeled by an equivalent circuit using the lumped parameter circuit elements such as capacitor, resistor, inductor and ideal transformers etc. Figure 2.7 shows the generic circuit for the transceiver (pulser and receiver) and its equivalent circuit using the lumped parameters circuit elements, proposed in [15].

Unlike [15], where we need more than one lumped parameter, Dang et al. in [19] and Schmerr et al. in [7] model the pulser by an equivalent voltage source and an equivalent impedance using Thevenin’s theorem, whereas, the receiver is modeled by an equivalent receiving impedance \( Z_r(\omega) \) and an amplification factor \( K(\omega) \). The advantage of this approach is that these parameters can be measured via a few electrical measurements, when quantitative validation is required.
2.5 Material Characterization

Accuracy of both the one-dimensional equivalent circuit model and three-dimensional finite element model depends on the accuracy of the material constitutive properties. For one dimensional piston model, we need only the longitudinal properties of the material. In contrast, a three dimensional finite element model needs material properties both in the longitudinal and transverse directions. Established protocols are crucial to characterize the materials if we want to use a model as a designing and a virtual prototyping tool. The properties available in the manufacturer specification sheets are often incomplete or sometimes there is a need to characterize a new material (with unknown properties) for certain requirements (e.g. making of a highly attenuating backing material). In the sequel, we discuss some established protocols available in the literature to characterize the active and passive materials.
2.5.1 Characterization of passive materials

The matching and backing layers in an ultrasonic transducer (see Figure 2.5) come in the category of the passive materials. For a one-dimensional model, we only need the longitudinal speed of sound and the attenuation constant, but for a three dimensional model we also need shear properties. An experimental setup that can be used to measure these properties and the method to find these properties are described next.

The required properties of a passive material can be found experimentally in a pitch-catch setup. In this setup, the acoustic waves are generated in a water bath using an ultrasonic transducer. The acoustic pressure generated by these waves is measured by a broad band hydrophone needle at some specified distance. The de-mineralized water should be used at the room temperature as the propagating medium. It is recommended to keep the distance between the hydrophone and the ultrasonic transducer as small as possible to avoid scattering effects. A three axis motion controller is used to place the hydrophone in front of the transducer. A rotator can be used to set the specimen at certain angles for the measurement of shear properties of the material. The received pressure waveform from the amplifier/coupler (attached to the hydrophone) is sampled by a digital oscilloscope. At first, we measure the pressure without a specimen, and then we put a specimen (whose properties need to be measured) between the hydrophone and the ultrasonic transducer as shown in the Figure 2.8. This experimental setup is used by many authors; see [20], [21],[22] and [23].

![Figure 2.8: Measurement of pressure through hydrophone with and without specimen](image)

When a specimen is placed between the transducer and hydrophone, the acoustic waves are attenuated due to the specimen and we received the waves earlier or
literature survey later depending on the speed of sound. In order to calculate the longitudinal speed of sound in the material and the attenuation constant, several authors discuss about the time domain techniques, see [20].

For the materials with high attenuation constant, the wave shape for the pressure wave changes dramatically as it transmitted through the material. In order to locate the equivalent points for finding the amplitude ratio is detrimental [21], using time domain signals. Other authors use frequency domain techniques developed by Sachse and Pao (1978), see [23]. Wang et al. in [23] explicitly mention the formulae to calculate the longitudinal speed of sound and attenuation constant, as given below:

\[ c_l = \frac{c_w}{1 + \frac{(\varphi - \varphi_w + 2\pi f \Delta t)c_w}{2\pi fd}} \]  
\[ \alpha_l = \alpha_w + \frac{1}{d}20\log_{10}\left(\frac{T_l A_w}{A}\right) \]  

In equations (2.18) and (2.19), \( \Delta t \) is the trigger delay of the signal, \( c_w \) is the speed of sound in water, \( f \) is the frequency, \( d \) is the thickness of the specimen, \( A_w, A, \phi_w \) and \( \phi \) are the amplitude and phase spectra, obtained by taking the Fourier transformation of the time domain pressure signal, without and with a specimen respectively. \( T_l \) is the product of the transmission coefficients for the interface of water to specimen and specimen to water:

\[ T_l = T_{w-s} T_{s-w} = \frac{4z_w z_l}{(z_w + z_l)^2} \]  

Here, \( z_w \) and \( z_l \) are the specific acoustic impedances of the water and specimen respectively. When the acoustic wave is incident at an angle on the object other than 0°, a shear wave is generated by the mode conversion effect [23]. Wang et al. in [23] mention the formulae to calculate the shear speed of sound and attenuation constant, as given below:

\[ c_s = \sqrt{\frac{c_w}{\sin^2(\theta_i) + \left[\frac{(\varphi - \varphi_w + 2\pi f \Delta t)c_w + \cos(\theta_i)}{2\pi fd}\right]^2}} \]  
\[ \alpha_s = \alpha_w \cos(\theta - \theta_i) + \frac{1}{d}20\log_{10}\left(\frac{T_s A_w}{A_s}\right) \]  

In equations (2.21) and (2.22), \( \theta_i \) is the incident angle, \( \theta \) is the refractive angle of shear waves and can be calculated from the Snell’s law. \( T_s \) is the product of the
transmission coefficients for the interface of water to specimen and specimen to water:

\[ T_s = T_{w \rightarrow s} T_{s \rightarrow w} = \frac{4z_wz_s}{(z_w + z_s)^2} \]  

(2.23)

Here, \( z_w \) and \( z_s \) are the specific acoustic impedances of the water and specimen (in shear position) respectively. In this way, we can characterize the passive materials for the model.

2.5.2 Characterization of active materials

The piezoceramic material is the active element in an ultrasonic transducer. The IEEE standard on piezoelectricity [24] facilitates for the measurement of material’s elastic, dielectric and piezoelectric properties by using certain geometrical shapes as shown in figure 2.9. These shapes are exclusively designed to isolate certain types of resonant behavior and to calculate the properties using a particular dominant mode. Furthermore, the standard [24] is based on the ideal lossless behavior of the active element. However, in practice, all the real materials exhibit losses. A refinement to this method has been developed by researchers at the Royal Military College of Canada to accurately determine a material’s properties [15]. The measurement of the active material properties involves the impedance measurement. This needs an experimental setup that can clamp a piezo element in the air with a minimum contact. To achieve this we can use needles to clamp the element as shown in figure 2.10. Furthermore, the setup required for the impedance measurement will be discuss in the following section.
2.6 Incremental Validation

In order to make use of the full scale model of an ultrasonic transducer propagating in an acoustic medium, the quantitative validation of the model with the experi-
mental results is crucial. Powell et al. in [18] describes an incremental "model-build-test" validation exercise to precisely model the complete ultrasonic transducer. The step by step procedure reported in the paper allows the user to tune the model at different step of manufacturing before the complete simulation of the measurement system. Abboud et al. in [15] discusses about the same technique to validate the model step by step.

During the manufacturing of the transducer, the piezoceramic element undergoes elevated temperatures and other complex manufacturing steps. This may cause partial depoling of the piezoceramic disc inside the transducer. Due to this fact, Powell et al. in [18] found 5% depoling of the piezo element and 10% reduced value of the speed of sound in the second matching layer with respect to the nominal material properties. Therefore, by correlating the experimental result with the simulation one can tune the material properties, which might be off from the nominal properties due to the measurement errors during material characterization or the complex manufacturing process.

The work presented in [18] is performed to validate the individual components of an array configuration of the transducers. This is completely applicable for a single transducer as well.

### 2.6.1 Step by step validation

A summary of the methodology adopted in [18] can be written in terms of different steps as given below.

1. Model the piezo ceramic element in air using the finite element method, based on the properties of the piezo ceramic material
2. Correlate the impedance curve of the model and real piezo element
3. Tune the material properties of the active element, if needed
4. Add a passive layer to the model, as per the order of the manufacturing process
5. Correlate the input impedance curve of the model and the newly manufactured configuration
6. Tune the material properties of the newly added passive layer, if needed
7. Goto step 3 until all the passive layers will be added to the model
8. Assume the depoling effect in order to correlate the impedance curve of the complete configuration
9. Now, correlate the input impedance of the complete transducer both in water
10. Put the transducer in the required package e.g in a catheter
11. Simulate the complete transducer in the required acoustic medium to analyze the output acoustic field
12. Modify the transducer geometry (if needed) and and go to step 11

2.6.2 Impedance measurement

Both for the material characterization and incremental validation process we need to measure the input impedance. A network analyzer of appropriate frequency range can be used for this purpose. Before measuring the impedance, it is necessary to calibrate the network analyzer for the open, short and known load (50 $\Omega$ usually used) termination conditions. Once the calibration is done, the input impedance of a transducer or piezo element can be measured in any acoustic medium. Figure 2.11 shows the measurement setup in this regard.

Figure 2.11: Measurement setup for the impedance measurement

2.7 Performance Measurement and Validation

Electrical measurements (such as the transducer impedance measurements) provide only an indirect measure of the acoustic performance [25]. To check the performance of a transduction device propagating in an acoustic medium, the following standard measurements and validation procedures can be adopted depending on the need of a designer.
2.7.1 Pulse-echo testing

One of the standard measures of the transducer acoustic performance is a pulse-echo test [25]. In this method, a transducer is excited (usually by an impulse) and the round-trip signal from an aligned target is obtained as shown in figure 2.12. The received signal is transformed into the frequency domain by a Fast Fourier transformation. This transformed signal is used to find the transducer bandwidth (at various levels e.g.-6dB) and the sensitivity. This testing is very common in practice, because the transducer is used in a pulse-echo mode for the imaging applications.

![Diagram of Pulse echo method](image)

**Figure 2.12: Pulse echo method**

2.7.2 Acoustic output measurement and beam plots

Hydrophones are a type of transducer used to measure pressure waves over an extremely wide bandwidth at an infinitesimally small spatial point [25]. A similar setup shown in figure 2.8 is used for measuring the acoustic pressure by the hydrophone. Medina et al. in [12] measure the acoustic pressure by a hydrophone at certain distances to validate their 2D axisymmetric transducer model.

A further extension of this measurement can be made by setting the transducer at a fixed position and align the hydrophone along the acoustic axis by translating in x, y, and z direction and rotation along the vertical axis. After alignment, the hydrophone is translated with a predefined resolution either in a horizontal plane or in a vertical plane to measure the acoustic pressure at different spatial points. Then a particular feature of the acoustic waveform, such as peak-to-peak voltage or signal amplitude, is determined and plotted. This kind of plot is referred as beam plot. It is common to normalize the beam plot to the largest value measured.
(for qualitative analysis) and present the data as a contour map with the contours representing decibel levels. Gutierrez et al. in [17] compared the normalized beam plot obtained by the simulation and experiment for the qualitative analysis of a transducer, as shown in figure 2.13

Figure 2.13: Normalized beam plot, measured for the model validation in [17]
Chapter 3

Conclusion and Recommendations

3.1 Conclusion

The field of medical diagnostic using ultrasonic transducers is moving forward at a rapid pace. Many designers employ one dimensional analytic models to predict the behavior of the ultrasonic transducer. However, due to certain limitations these models are not adequate to understand the entire performance of an ultrasonic transducer. Nowadays, finite element modeling is being adopted in the design of ultrasonic transducers. The objective is to get more realistic transducer simulations and to accelerate the design process.

The purpose of this literature study is to serve as a primer for modeling an ultrasonic transducer using the commercial software package COMSOL. The report elaborates various issues that arise during the finite element modeling of an ultrasonic transducer. The present study starts with basic transducer configuration and design guidelines. The breadth of this report focuses on the background and practical modeling issues that affect simulation strategies. The dependence of the simulation results on accurate model parameters including material properties, adequate mesh resolution, realistic boundary conditions and appropriate geometrical simplifications are discussed. Various standard procedures to characterize material properties for both active and passive materials are presented. As modeling is the part of transducer design, so it is not independent of the experimental validation. In addition, a detailed experimental validation procedure along with the performance measurement techniques is elaborated.
3.2 Recommendations

The study presented in this report highlights various aspects of finite element acoustic modeling. Although these issues are not all-encompassing, it provides a basic foundation to a modeler or a designer for modeling an ultrasonic transducer using finite element methods.

The modeling process involved in this report is not only useful for the ultrasonic transducer used in cardiac application, but, it is applicable to any ultrasonic measurement system for diagnostic and inspection. Such a system can be a non-destructive evaluation system for an aerospace structure or underwater pipe line inspection or any medical diagnostic system. New horizons need to be found for the application of ultrasonic transducers either by using a single ultrasonic transducer or in array. In future, the need of ultrasonic transducers in medical diagnostics can be more demanding due to the reduction in size of ultrasonic transducers to micro scale level.
References


