The determination of the pressure-viscosity coefficient of two traction oils using film thickness measurements

Harry van Leeuwen

Abstract—The pressure-viscosity coefficients of two commercial traction fluids are determined by fitting calculation results on accurate film thickness measurements, obtained at a wide range of speeds, and different temperatures. Film thickness values are calculated using a numerical method and approximation formulas from twelve models. It is concluded that, to assess the pressure-viscosity coefficient of these traction fluids, the Hamrock and Dowson approximation formula, and derivatives: Hamrock et al. and Chittenden et al., are the best choice, having an inaccuracy in between (-12%, +7%). These equations have been used far outside the regime of the numerical data where they were based on.

I. INTRODUCTION

With ever increasing power densities, the pressure dependency of fluid viscosities is becoming more and more important. An example is the high pressure viscosity of diesel fuel with common rail systems, where maximum pressures will soon reach 0.3 GPa. This is expressed in the so-called pressure-viscosity coefficient $\alpha$. This pressure-viscosity coefficient is also a very important parameter in film formation and traction, and is a decisive parameter in the proper functioning of traction fluids, which serve as lubricants in continuously variable transmissions (CVT’s) where maximum pressures may exceed values far in excess of 1.0 GPa.

In an earlier paper [1] a method was introduced to employ very accurate film thickness measurement results in determining the pressure-viscosity coefficient of an arbitrary fluid. In this method, the most recent film thickness model, and the believed most accurate one to date, was used: the Moes [2] central film thickness formula for elliptical EHL (elastohydrodynamically lubricated) contacts. The measurements were carried out on a common interferometric device [3], [4] and is an alternative to high pressure viscometers [5], [6]. This alternative is interesting, since many film thickness measurement systems can be found throughout the world, whereas only a few institutes have high pressure viscometers.

Johnston et al. [3] were among the firsts to employ a film thickness measurement device as a viscometer. Later, Cooper and Moore [7], Gunsel et al. [8], and recently Bantchev and Biresaw [9] and Pensado et al. [10] reported on a determination of the pressure-viscosity coefficient through film thickness measurements. In all these articles Hamrock and Dowson’s [11] film thickness equation was chosen, without asking oneself whether this approximation would be admissible or appropriate. Bair [12] is questioning the classical EHL approach and states that a film based assessment of $\alpha$ is deemed to fail, since the rheological model is in error, leading up to worst case errors of over 40 per cent.

In [1] a comparison is made between 11 film thickness models, all based on classical EHL assumptions, to find which one was the best to find the value of $\alpha$. The results were satisfying for the test lubricant, a mineral oil, HVI60. The best film thickness model was the one provided by Chittenden et al. [13], with a maximum error of about 10% in the estimate. Nevertheless, the test fluid had a relatively small pressure-viscosity coefficient and correspondingly small film thickness values. There are good reasons to assume that the Chittenden et al. model will perform less well for lubricants having high $\alpha$ values, like with traction oils, and that the Moes model will yield better results. On the other hand, all the current film thickness models assume Newtonian behavior, a requirement which often is not met in the case of traction oils.

In this paper estimates derived by application of many film thickness models are compared with the known effective pressure-viscosity coefficient of two traction oils, Santotrac S50 and S70, to examine whether a film thickness based assessment is allowable. S70 is also treated in [14], where it was again concluded that the Chittenden et al. model performed best. S50 is known for its considerably lower film thickness in experiments, when compared with a Hamrock and Dowson prediction [5], and has not been investigated in this way yet.

II. A FILM THICKNESS BASED ASSESSMENT OF THE PRESSURE VISCOSITY COEFFICIENT

Methodology

Film thickness measurements are carried out for two traction fluids, at 40, 60 and 80 °C, and 4, 20 and 50 N load, corresponding to 0.30, 0.50, 0.70 GPa Hertzian...
contact pressure. Load and temperature were kept constant during a series, under zero spin and slip, while the entrainment speed was varied in proportional steps from 0.01 until about 3 m/s.

First the experimental film thickness values are compared with numerical results, obtained with classical EHL computations. All film thickness approximation equations are based on numerical data from classical EHL. Therefore, it can be expected that the estimates obtained with these equations are only acceptable if the numerical results for the film thickness are sufficiently close to the experimental values.

Twelve film thickness model equations from the literature are fitted on these experimental results, by adapting the pressure-viscosity coefficient until a minimum in the root mean square error is found. These values are compared with the known values.

**Film thickness measurements**

The measurements were performed using an EHL Ultra Thin Film (UTF) Measurement System [4]. This system allows measurements down to 1 nm with an inaccuracy of less than 1 nm. The device is operated in central film thickness measurement mode, under pure rolling, see Fig. 1.

![Fig. 1. Schematic of the PCS Instruments Ultra Thin Film measurement system.](image)

The measurement procedure is as follows.

1. First both the ball and glass disc are cleaned in toluene in an ultrasonic bath during at least 10 minutes, next with ethanol, air dried, and mechanically cleaned using lens tissue;
2. The first measurement was performed after at least 30 minutes, to ascertain that thermal equilibrium was established;
3. Five readings were averaged and if the standard normalized deviation was smaller than 1, this average value was accepted.

**Lubricant data**

The lubricants used were Santotrac S50 and S70 oil. Their pressure-viscosity coefficients have been determined on a high pressure viscometer and are listed in Table 2.

| Table 2 Properties of Test Fluids: Santotrac S50 and S70 |
|---------------------------------|-------|-------|-------|
| Temperature (°C) | 40 | 60 | 80 |
| Dynamic viscosity \( \eta_0 \) (Pa·s) | 26.81*10^{-3} | 12.68*10^{-3} | 6.92*10^{-3} |
| Pressure-viscosity coefficient \( \alpha^* \) (GPa^{-1}) | 28.2 | 23.0 | 19.3 |

| Santotrac S70 |
|---------------------------------|-------|-------|-------|
| Dynamic viscosity \( \eta_0 \) (Pa·s) | 97.65*10^{-3} | 33.4*10^{-3} | 15.5*10^{-3} |
| Pressure-viscosity coefficient \( \alpha^* \) (GPa^{-1}) | 36.8 | 30.0 | 25.0 |

These data are close to other data from the literature, see [16], [17].

**Traction measurements**

The coefficients of traction of the test fluids were determined with a PCs Instruments MTM Mini Traction Machine, under 0.50, 1.00 and 1.25 GPa pressure, 40, 60 and 80 °C, 0.25 – 2.00 m/s entrainment speed, and 0 – 10% Slide to Roll Ratio (SRR).

The UTF measuring device does allow for traction...
curve measurements, but is less suited. In traction mode, the ball driven through a horizontal shaft, see Fig. 1, causing spin in the contact, as is also discussed by Willermet et al. [18]. Traction is very sensitive to spin. In an MTM the ball axis is skewed to compensate for spin. Some results are shown in Figs. 2a and 2b, to ascertain that their traction values are in a range comparable to [18].

(3) analytical formulas, often for a specific regime:
- Archard and Cowking [25];
- Hooke [26];
- Sutcliffe [27]

(4) general formulas, based on (1) and (3), containing asymptotic formulas for the Johnson [28] lubrication regimes:
- Venner [29];
- Nijenbanning, Venner and Moes [30];
- Venner and Lubrecht [19], and
- Moes [2]

All these formulas are described and extensively discussed in [1] and will not be recited here. Most of them have been developed for elliptical contacts, but will be used for circular contacts here.

III. RESULTS

Film thickness measurements

First the measured central film thickness values will be compared with numerical predictions. The software used is from Venner [31]. Figure 3a shows the central film thickness results for S50 at 0.50 GPa in an entrainment speed range between 0.01 – 3 m/s.

Fig. 3a. Central film thickness for Santotrac S50, at 0.70 GPa and variable rolling speed. Legend: measurements ○ at 40 °C, Δ at 60 °C, and □ at 80 °C, calculations — *—* (log scales; top to bottom: 40, 60, and 80 °C).

It is seen that the numerical results match the experimental data for 40 and 60 °C very well, but for 80 °C the experimental values were found to be considerably larger than the numerical data.

Figure 3b presents the central film thickness for S70 at 0.50 GPa and an entrainment speed range between 0.01 - 2.2 m/s.

Fig. 2b. MTM tractions curves for Santotrac S70, at 1.25 GPa and 2 m/s rolling speed. From top to bottom: 80 °C, 60 °C, and 40 °C.

In comparison to [18] S50 shows 4-5% a higher traction coefficient, which can be attributed to the rolling speed of 2.0 m/s, while in the latter case 5.08 m/s was selected.

Numerical film thickness calculations

To check whether the experimental values can be described by classical EHL theory, numerical calculations will be performed, see Venner and Lubrecht [19], for many of the experimental conditions. They are subject to assumptions from classical EHL theory, as isothermal flow, Roelands [20] viscosity-pressure behavior, and a density-pressure behavior as in Dowson and Higginson [21]. All film thickness approximation formulas are essentially based upon assumptions like these or more restrictive ones, see [1]. In addition, all of these formulas postulate fully-flooded conditions.

Film thickness approximation formulas

Several types of film thickness equations were used, which are categorized as follows:

(1) interpolation formulas, usually power law expressions, fitted on numerical data only:
- Hamrock and Dowson [11];
- Hamrock et al. [22];
- Chittenden et al. [13];
- Evans and Snidle [23],

(2) interpolation formulas, usually power law expressions, fitted on numerical and experimental work:
- Greenwood¹ [24]

¹ Greenwood refrained from suggesting any film thickness formula, but this approximate formula can be derived from his data
The central film thickness for S70 is a factor of 2.8 higher than for S50, as is witnessed by Fig. 3c.

The pressure-viscosity coefficient

The estimates for the 6 values of the pressure-viscosity coefficients as given in Table 3A at 20 N (0.50 GPa) load.

TABLE 3

(A) Overview of pressure-viscosity estimates for Santotrac S50 and S70

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(B) Deviation in (A) from the target values

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average deviation: 40.2% -2.1% -2.3% 7.1% 1.1% 20.5% 24.5% 6.1% 15.2% 18.6% -18.3%
### IV. DISCUSSION

Table 3b lists the deviations from the target values, 3c the standard deviation in the central film thickness prediction (by using the obtained value of $\alpha$), and 3d shows the correlation between the calculated and the measured values.

The deviations from the target values can be as large as 50% for the equation from [25], and $\pm 25\%$ [26] or $\pm 25\%$ [27], if the S60 series at 80°C is omitted. The best predictors are in group (1), except Evans and Snidle [23]. Group (2) and (3) give unsatisfactory results. Group (4) is not so good in the estimates, it is consistently 15-18% too low, but is very good in the standard deviation and the correlation.
equation is always employed, and this apparently is a good choice in this case, just as good as the Hamrock et al. or the Chittenden et al. equations. It must be noted that the group (1) formulas are used under conditions far beyond those of the numerical results where they are based upon, see the Appendix. In this appendix the advantage of using nondimensional groups is also illustrated. Group (4) equations are based on asymptotic solutions, and should be appropriate to deal with these conditions. Indeed, group (4) equations yield the asymptotic solutions, and should be appropriate to deal with these conditions. In the results reported here all group (4) equations conclude that the Moes fits are close to their numerical estimate for \( \alpha \). This is confirmed by Chaomleffel et al. \[33\]. Several authors note that S50 shows shear thinning \[5\], resulting in a much lower measured film thickness than calculated. However, the results in this study do not corroborate this finding. This could be due to the pure rolling of the ball over the disc, and shear thinning may immediately manifest itself if a little slip is encountered.

The numerical results show that a classical EHL approach is still admissible. If there were a perfect curve fit of these numerical results, this fit would yield a much better value for the pressure-viscosity coefficient – but this equation does not seem to exist yet. It must be concluded that there is room for improvement.

V. CONCLUSIONS

(1) The central film thickness in a circular EHL contact was measured for two traction fluids, for a wide range of conditions. Twelve approximation formulas were employed to assess the value of the pressure-viscosity coefficient, and to find which one performs best.

(2) The numerical calculation results were close to the measured values, suggesting that classical EHL conditions are valid, and that the use of the approximation formulas makes sense.

(3) In this study, the Hamrock and Dowson \[11\], Hamrock et al. \[22\], and Chittenden et al. \[13\] formulas yielded the best values for the pressure-viscosity coefficient. Applying them under the conditions of the tests in this report, means that they are used far beyond the conditions where they were fitted upon.

(4) All existing approximation formulas have drawbacks. A better approximation formula is still sought after.

APPENDIX: NON-DIMENSIONAL GROUPS

After a long debate in the 70’s, it was concluded that 3 non-dimensional groups suffice in describing the central or minimum film thickness in highly loaded line contact elliptical EHL contacts. They were first defined by Blok \[34\], and adapted slightly by Moes \[35\]. For elliptical contacts, 4 groups are needed, defined as:

\[
H = \left( \frac{h}{R_e} \right)^{\frac{1}{2}} \left( \frac{E_r R_e}{2 \eta_0 \alpha} \right)^{\frac{1}{2}} \text{ film thickness parameter}
\]

\[
M = \left( \frac{F}{E_r R_e^2} \right)^{\frac{1}{2}} \left( \frac{E_r R_e}{2 \eta_0 \alpha} \right)^{\frac{1}{2}} \text{ load parameter} \quad (1)
\]

\[
L = \left( \frac{2 \eta_0 \alpha}{E_r R_e} \right)^{\frac{1}{2}} \text{ lubricant parameter}
\]

\[
\omega = \frac{R_i}{R_e} \text{ radii of curvature ratio}
\]

This minimum set of nondimensional groups allows more convenient representations in diagrams and better curve fitting of numerical data than with a larger set.

To acknowledge the ellipticity of the contact, the effective radii of curvature ratio (crowning ratio) \( \omega \), had to be added to the original set \[35\] for line contacts. In these equations the following symbols have been used.

For the geometry: \( h \) film thickness (m); \( R_e \) the effective radius of curvature in the direction of entrainment (m); \( R_i \) the effective radius of curvature perpendicular to the direction of entrainment, or transverse (m). For the operational conditions: \( F \) load (N); \( \bar{U} \) the entrainment or average tangential speed of the surfaces (m/s). For the materials: \( E_r \) the equivalent modulus of elasticity of the solids (N/m²); \( \eta_0 \) the fluid dynamic viscosity under ambient conditions (Ns/m²); \( \alpha \) the fluid pressure-viscosity coefficient (m²/N).

In (1) the equivalent modulus \( E_r \) is found from the E moduli \( E_{11}, E_{22}, \) and Poisson coefficients \( \nu_{11}, \nu_{22}, \) of the two solids 1 and 2 through:

\[
E_r = \left[ \frac{1}{2} \left( \frac{1 - \nu_{12}^2}{E_{11}} + \frac{1 - \nu_{21}^2}{E_{22}} \right) \right]^{-1} \quad (2)
\]

whereas in a case that the two bodies both have a curvature radius in the same plane, the reduced or effective radius is defined as

\[
R_i = \left[ \frac{1}{R_{11}} + \frac{1}{R_{22}} \right]^{-1} \quad (3)
\]

where \( i \) means: in entrainment ‘e’ or transverse ‘t’ direction.

Transformations between different sets of nondimensional groups are listed in \[1\]. All nondimensional parameters appear linearly in (1), including the nondimensional...
pressure-viscosity coefficient or lubricant parameter \( L \).

The nondimensional film thickness \( H \) can now be mapped as a function of the non-dimensional load \( M \) and the nondimensional pressure-viscosity coefficient \( L \), for a certain value of the ellipticity parameter \( \omega \): \( H = H(M, L, \omega) \). For circular contacts, as employed in many investigations, \( \omega = 1 \), yielding \( H = H(M, L) \). This permits a representation in a diagram where \( L \) is the parameter, and \( H \) is depending on \( M \), see Fig. 4.

All recent EHL work follows this representation. However, these data can easily be represented in a more convenient map, see Fig. 5.

Here the independent variables are \( M \) and \( L \). This has several advantages, viz. that the threshold from rigid to elastic behavior of the solids can easily be drawn, and that the common Johnson regimes of lubrication can be distinguished (see [28]). The contours in this diagram are based on the Moes [2] film thickness equations, so the contours’ accuracy depends on the accuracy of the Moes equations.

The marks in Figs. 4 and 5 show the data points where Chittenden et al. [13], and Hamrock and Dowson [11] performed their numerical simulations. Due to their very high pressure coefficient \( \alpha \), synthetic traction oils will reach high \( L \) values, much higher than when normal oils are employed, thus allowing measurements at regimes far beyond those of the mineral oil employed in [1], see Fig. 6,

![Fig. 4. Cross sectional diagram of theoretical nondimensional film thickness \( H = H(M, L) \) in a circular EHL contact. Legend: \( \bullet \) = VR/VE transition, \( \bullet \) = data from Hamrock and Dowson [10], \( \Delta \) = data from Chittenden et al. [13]](image)

![Fig. 6. Contour plot showing the measurement conditions in this report. Load 20 N. Legend: \( \blacktriangle \) S50 at 60°C; \( \blacktriangle \) S50 at 60°C; \( \blacktriangle \) S50 at 80°C; \( \blacktriangle \) S70 at 80°C; \( \blacktriangle \) S70 at 80°C; \( \blacktriangle \) S70 at 60°C; \( \blacktriangle \) S70 at 40°C.](image)

\[\text{ACKNOWLEDGMENT}\]

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