Production error analysis for a line of manufacturing machines, variable structure control approach

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Abstract. Nowadays, development of wide variety of different types of products requires more and more complex configurations of production networks out of modern manufacturing industries. Thus, efficient methods for control of complex manufacturing networks are required. In this paper we introduce such a control method for a line of manufacturing machines. The main objective of our method is to guaranty that the number of produced products follows the current production demand. First a general idea of the variable structure control method is given thought the simple case of one manufacturing machine. Then, a flow model of a line of machines with variable structure control is presented. Consequently, the obtained results on the uniform ultimate boundedness of the production errors of each machine in the line are discussed. Performance and robustness issues of the closed-loop flow line model are illustrated in numerical simulations.

Keywords: manufacturing systems, tracking systems, discrete time systems, boundary conditions, variable structure control

1 Introduction

Nowadays, high market competition in brands and varieties in type of products confronts the manufacturing industries with a question of how to keep efficient track of the product demand. In addition, due to the recent events in the global economy keeping a big amount of over produced products in storage for the future distribution is becoming a risky strategy.

In this paper we present recent results of research on control of complex manufacturing networks. The goal of this research is to develop efficient methods for control of complex manufacturing networks, so that the number of produced products follows the current demand. This control method does not require the knowledge of the future demand and is capable of taking real time control decisions. Particular attention is paid to the constraints present in the network, such as capacity and buffer limitations. Each machine in the network has a restricted number of products that this
machine can produce in a fixed period of time, known as the capacity constraint. The content of the buffer between two machines is given by the difference between the total number of products produced by the upstream machine and the total number of products produced by the downstream machine. Considering that a manufacturing line has a unidirectional product flow implies that the buffer content can never be negative, e.g. the downstream machine cannot produce more than the upstream one.

In particular, this paper contains an explicit description of flow model (see, e.g., [1], [2], [3], [4]) for a line of manufacturing machines, where a variable structure control is introduced as an alternative control technique, in order to give a solution to the product demand tracking problem. Due to the fact that in practice all the machines in the line are working simultaneously and the control actions are executed in discrete time, the $\chi$ language (see, e.g., [5], [6]) and MATLAB® are used in order to simulate the manufacturing process.

2 One Machine

2.1 Flow Model

The basic idea of our method can be understood through the analysis of one manufacturing machine. A simple manufacturing machine can be interpreted as an integrator (see, e.g., [1], [7]), where the cumulative number of finished products is obtained through the integral of its production rate. Considering this, the flow model of one manufacturing machine in discrete time is defined as

$$y(k + 1) = y(k) + u(k) + f(k), \quad (1)$$

where $y(k) \in \mathbb{R}$ is the cumulative output of the machine, $f(k) \in \mathbb{R}$ is an unknown external disturbance, and $u(k) \in \mathbb{R}$ is the control signal.

Taking into account that there is always sufficient raw material to feed the machine, the control aim is to track the non-decreasing cumulative production demand. Thus, we define the production demand through $y_d(k) \in \mathbb{R}$, which is given by

$$y_d(k) = y_{d0} + v_d k + \varphi(k). \quad (2)$$

Here, $y_{d0}$ is a positive constant that represents the initial production demand, $v_d$ is a positive constant that defines the average desired production rate, and $\varphi(k) \in \mathbb{R}$ is the bounded market fluctuation that is imposed on the linear demand $v_d k$, where constant $k$ represents the unit of time.

In order to give a solution to this tracking problem we propose the following control input

$$u(k) = A \text{sign}_+(\varepsilon(k)). \quad (3)$$

Here $A$ is a positive constant that represents the processing speed of the machine, $\text{sign}_+(\varepsilon(k)) = (1, \text{ if } \varepsilon(k) > 0) | 0, \text{ otherwise}$, and $\varepsilon(k) \in \mathbb{R}$ is the output
production error with respect to the demand. This production error is given by
\[ \varepsilon(k) = y_d(k) - y(k), \]
where \( \varepsilon(k - 1) + \varepsilon(k) \) along the solutions of \( \varepsilon(k) \) is given by
\[ \varepsilon(k + 1) - \varepsilon(k) = v_d + \Delta \varphi(k) - \text{sign}(\varepsilon(k)) - f(k), \tag{4} \]
with \( \Delta \varphi(k) = \varphi(k + 1) - \varphi(k) \).

Now, from (4) it is evident that in order to guaranty a proper demand trajectory tracking the product demand cannot be higher than the machine processing speed, which in this case is \( \Lambda \) lots per time unit. Thus, let us consider that \( \Delta \varphi(k) - f(k) \) from (4) is bounded by
\[ \alpha_1 < \Delta \varphi(k) - f(k) < \alpha_2, \quad \forall k \in \mathbb{N}, \tag{5} \]
where \( \alpha_1, \alpha_2 \) are some constants that satisfy
\[ \alpha_2 < \Lambda - v_d, \tag{6} \]
\[ \alpha_1 > -v_d. \tag{7} \]

Here, by inequality (6) we state that the current demand cannot be bigger or equal to the machine production speed and by inequality (7) that under no circumstances the customer demand can take a negative value. This last consideration is made based on the logical assumption that once the product is made it can never return in to the production line. Thus, from (5), (6), and (7) the following condition holds
\[ 0 \leq v_d + W(k) < \Lambda, \tag{8} \]
where \( W(k) = \Delta \varphi(k) - f(k) \), which is considered as an unknown bounded disturbance affecting the machine performance.

### 2.2 The Main Result

In this paper we omit the detailed analysis of the tracking error behavior of the flow model (4). None the less, we formulate the main result in the following manner. Let the discrete time system defined by (1) with control inputs (3) satisfy condition (8), then all the solutions of (4) are uniformly ultimately bounded by
\[ \lim_{k \to \infty} \sup \varepsilon(k) \leq v_d + \alpha_2, \tag{9} \]
\[ \lim_{k \to \infty} \inf \varepsilon(k) \geq v_d + \alpha_1 - \Lambda. \tag{10} \]

Basically, in order to follow the product demand, variable structure controller \( u(k) = \text{sign}(\varepsilon(k)) \) is introduced to the flow model of one machine. The production error \( \varepsilon(k) \) of a single machine is defined as the difference between the cumulative demand \( y_d(k) \) and the cumulative number of products produced up to this moment \( y(k) \). If the tracking error is positive, meaning that more products are required, then the controller activates the machine at its full production speed \( \Lambda \) or else if the tracking error is negative then the controller simply stops the machine and does nothing until the error is positive again.
3 A Line of Machines

3.1 Flow Model

For the flow model of a manufacturing line the previous control strategy is modified with respect to the number of buffers and machines present in the line. A new limitation such as desired buffer content is considered in the model.

The flow model of the manufacturing line is defined as

\[ y_j(k+1) = y_j(k) + \beta_j(k), \quad j = 2, \ldots, N, \]  

where \( y_j \) is the output of the machine \( j \), \( w_j(k) = y_{j-1}(k) - y_j(k) \) is the buffer content of the buffer \( j \), \( \beta_j(k) = u_j(k) + f_j(k), \forall j = 1, \ldots, N \), \( f_j(k) \) is the external disturbance affecting the machine \( j \) e.g. production speed variations, undesired delay), \( u_j(k) \) is the control input of the machine \( j \) and \( \text{sign}_{\text{buff}} \left( w_j(k) - \beta_j(k) \right) = (1, \text{if } w_j(k) - \beta_j(k) \geq 0 \mid 0, \text{otherwise} \).

In order to give a solution to the demand tracking problem we propose the following control inputs:

\[ u_j(k) = \Lambda_j \text{sign}_+ \left( \epsilon_{j+1}(k) + \left( w_{dj+1} - w_{j+1}(k) \right) \right), \quad \forall j = 1, \ldots, N - 1 \]  

where \( w_{dj+1} \) is the desirable buffer level of buffer \( j + 1 \), \( \epsilon_{j+1}(k) \) is the tracking error of machine \( j + 1 \), and \( \Lambda_j \) is the processing speed of machine \( j \).

The tracking error of each machine is given by

\[ \epsilon_j(k) = \epsilon_{j+1}(k) + (w_{dj+1} - w_{j+1}(k)), \]  

\[ \epsilon_j(k) = y_d(k) - y_j(k) + w_{dj+1} + \cdots + w_{dN}, \quad j = 1, \ldots, N - 2, \]  

\[ \epsilon_{N-1}(k) = y_d(k) - y_{N-1}(k) + y_N(k), \]  

\[ \epsilon_{N-1}(k) = y_d(k) - y_{N-1}(k) + w_{dN}, \]  

\[ \epsilon_{N-1}(k) = \epsilon_N(k) + (w_{dN} - y_N(k)), \]  

\[ \epsilon_N(k) = y_d(k) - y_N(k). \]  

It follows from (17) that the error of machine \( N \) is defined exactly as for the single machine case. The buffer restriction, as seen from (12), is the only difference in the flow model of machine \( N \) with the flow model of (11). For (15), (16) the new considerations are applied for the tracking error of each machine \( j \), where \( j = \)}
Thus, tracking error $e_j(k)$ depends on number of produced products $y_j(k)$ with respect to current demand $y_d(k)$ plus the desired buffer content of each downstream buffer $w_{dj+1}$. This means, that every upstream machine needs to manufacture $w_{dj+1}$ lots more than the downstream one. The $w_d$ is introduced in order to prevent downstream machines from lot starvation, e.g. in case of a sudden growth of the product demand.

It is also important to take into account that the control action is decentralized throughout the network. In other words the control action of each machine in the line only depends on the tracking error of its neighboring downstream machine (except for machine $N$, which depends directly on cumulative demand input) and the current buffer content of its upstream buffer. That gives our flow model an extra robustness with respect to the undesired events such as temporal machine setup or breakdown.

### 3.2 The Main Result

For further analysis, let us rewrite the flow model (11), (12) in a closed-loop with (13), (14) as

$$\Delta e_j(k) = v_d + \Delta \rho(k) - f_j(k) - A_j \text{sign}_+ \left( e_j(k) \right),$$  \hspace{1cm} (18)

$$\Delta e_j(k) = v_d + \Delta \rho(k) - f_j(k) - A_j \text{sign}_+ \left( e_j(k) \right) \text{sign}_{\text{buff}}(w_j(k) - \beta_j(k)),$$  \hspace{1cm} (19)

where $\Delta e_j(k) = e_j(k+1) - e_j(k), \forall j = 1, \ldots, N$.

Here, it is important to notice that in this model we consider that each machine in the line has its own processing speed of $\beta_j$ lots per time unit and the buffer condition is considered as

$$w_j(k) \geq \beta_j(k), \ \forall j = 2, \ldots, N.$$  \hspace{1cm} (20)

This condition means that if machine $j$ is activated by control logic in time $k$ then in order for this machine to start working the content of buffer $j$ needs to be at least $\beta_j$ lots.

Now, let us assume that $w_{dj}$ satisfy the following inequality

$$w_{dj} \geq \beta_j(k) + v_d + \alpha_2, \quad \forall k \in \mathbb{N}, j = 2, \ldots, N,$$  \hspace{1cm} (22)

and that conditions (5), (6), (7) and (8) are satisfied for each machine $j$. Then, we have proven that for manufacturing line defined by (18), (19) all the solutions of (18), (19) are uniformly ultimately bounded by

$$\limsup_{k \to \infty} e_j(k) \leq v_d + \alpha_2,$$  \hspace{1cm} (23)

$$\liminf_{k \to \infty} e_j(k) \geq v_d + \alpha_1 - A_1.$$  \hspace{1cm} (24)
In consequence, for the buffer content \( w_j(k) \) of each buffer \( j \) defined by (15), (16), considering the obtained tracking error bounds (23), (24), it holds that

\[
\lim_{k \to \infty} \sup w_j(k) \leq \Lambda_j - \alpha_j + \omega_j.
\]

(25)

The complete analysis of the closed-loop system is not considered in this paper. Instead, numerical simulations are presented to support the present development.

4 Simulation Results

4.1 One Machine Case

Simulation results for one manufacturing machine, driven by the variable structure regulator (3), are presented in this section. For all the simulation results in this section the machine processing speed was fixed to 5 lots per time unit.

Figure 1 show tracking error and output response of the machine to the nonlinear demand growth. Here, product demand \( y_d(k) = 35 + 2k + \sin(5k) \). The resulting tracking error is bounded by \( 3 > \varepsilon(k) > -4 \) lots, which satisfy (9), and (10). It can be observed from figure 1 (right) that, given initial demand of 35 lots and initial output of 0 lots the machine manages to reach the demand trajectory in 12 time steps.

![Fig. 1. Tracking Error and Demand vs. Output, with \( v_d = 2 \) lots per time unit, \( y_d = 35 \) lots and \( \phi(k) = \sin(5k) \).](image)

4.2 Four Machines Case

Simulation results for a line of 4 manufacturing machines, driven by the variable structure regulators (13), and (14) are presented in this section. Here, for all the simulations the processing speed for each machine was set to \( \Lambda_j = 8,10,7,6 \) (lots per time unit), with \( j = 1,\ldots,4 \) and the desired buffer content of each buffer was selected considering (22) as \( w_{dj} = 20,14,12 \) (lots), with \( j = 1,\ldots,4 \). The tracking error of each machine in the line is depicted in figure 3.
Here, the initial conditions \((y_{d0}, y_1(0), y_2(0), y_3(0), y_4(0)))\) were set to the zero value. After the first 24 time steps, as shown in figure 2 (right), the system reaches its steady state. The tracking errors are maintained inside \([-2.5]\) lots for machine 1, \([-4.1]\) lots for machine 2, \([-1.5]\) lots for machine 3, and \([0.5]\) lots for machine 4, which satisfy (23), (24). The buffer level of each buffer is depicted in figure 2 (left). Considering that given desired buffer level of each buffer satisfies (22), the buffer content of each buffer satisfies the upper bound restriction (25).

![Fig. 2. Buffer Content and Demand vs. Output, with \(v_d = 5\) lots per time unit.](image)

![Fig. 3. Tracking Errors, with \(v_d = 5\) lots per time unit.](image)

In conclusion the simulation results reflect the expected flow model behavior for all the product demands that are given in this section. These product demands were selected in order to test the model behavior inside the boundary of the given capacity condition (8).

5 Conclusion

A variable structure control technique has been proposed in order to give a solution to the demand trajectory tracking problem. The bounds for the customer demand
tracking error were obtained for the case of one manufacturing machine working under unknown bounded perturbations. Also, developed results provide the bounds for the customer demand tracking errors of each machine in the line, as well as, the upper bound for the buffer content of each buffer in the line of machines considering that each machine can have a different processing speed. Simulations were presented and discussed in order to support these analytical results. The simulation results reflected effectiveness and robustness of the flow models.

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**References**