FEEDBACK STABILIZATION OF TRANSITION BOILING STATES

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ABSTRACT
A nonlinear one-dimensional heat-transfer model for pool boiling systems is considered. The model involves only the temperature distribution within the heater and models the heat exchange with the boiling medium via a nonlinear boundary condition imposed at the fluid-heater interface. This compact model is employed for the design and analysis of a control strategy for the stabilization of unstable states in order to improve cooling applications based on boiling heat transfer. Therefore, a state feedback controller is implemented that regulates the heat supply as a function of the system’s internal state. Simulations of the nonlinear closed-loop system expose the state feedback controller as a viable option for the rapid stabilization and regulation of pool boiling systems at an unstable equilibrium point.

KEYWORDS: Pool boiling, stabilization, numerical simulation, transition boiling

INTRODUCTION
Further development in cutting-edge technologies becomes increasingly reliant upon the ability of massive heat removal. As a result, cooling methods based on boiling heat transfer are emerging as novel cooling technique, since liquid boiling affords cooling capacities substantially beyond those of conventional methods (air/liquid cooling) [1]. Pool boiling may serve as physical model for cooling applications using boiling heat transfer and (controlling) its dynamical behavior is the principal subject of investigation here.

Pool boiling refers to boiling heat transfer by natural convection and admits two stable modes: nucleate (liquid-rich) boiling at “lower” temperatures and film (vapor-rich) boiling at “higher” temperatures. Nucleate boiling is the most efficient boiling mode and, consequently, the desired state in cooling applications. Nucleate and film boiling are connected through transition boiling. Transition occurs if the heat generation exceeds the so-called “critical heat flux” (CHF) and thermal equilibrium is possible only in the film-boiling regime [2]. As a result, the temperature rises significantly, and the cooling capacity collapses. Hence, optimal cooling performance is a trade-off between close proximity to CHF (efficient nucleate boiling) and a safety margin.
(prevention of transition). Current phase-change cooling schemes require a relatively large safety margin due to two key limitations: (i) high uncertainty in predicting CHF and system dynamics; (ii) the inability to actively respond to fluctuating cooling demands due to the passive working principle [1,3]. Therefore control strategies must be developed to safely facilitate boiling heat transfer close to CHF under dynamic operating conditions. In [4] such a control strategy is developed for the compact pool boiling model introduced and analyzed in [5,6]. This model describes the pool boiling dynamics entirely in terms of the temperature field within the heater. Thus the system reduces to a heat-transfer problem for the heater with a nonlinear heat-flux relation at the fluid-heater interface. For brevity, this study is restricted to uniform boiling states on the fluid-heater interface and thus utilizes the 1D simplification of the compact model from [5,6]. Controllers have to date been designed specifically for scientific boiling-curve measurements via the 1D approach by [7–10]. However, these controllers are not able to stabilize the system for a wide range of combinations of heater properties and heating conditions, e.g. for thick heaters and/or heaters with low thermal conductivity, see [4]. Furthermore, they can not respond rapidly to dynamic operating conditions. The present study seeks to overcome these limitations by further designing the control strategy developed in [4]. The aim is to design a state feedback controller that can regulate the system for a wider range of heater properties and heating conditions. Furthermore, the controller is defined in such a way that the closed-loop system exhibits user-defined properties. The following controllers are considered. Controllers which (i) quickly respond to disturbances and initial perturbations, (ii) slowly respond to these perturbations, such that low input values are required and (iii) result in sub or super critical damped systems. A sub critical damped system overshoots its goal several times, before it reaches equilibrium, when starting from an initial perturbation. The oscillations are the result of complex poles near the imaginary axis. In a supercritical damped system such oscillating behavior is not observed.

**MODEL AND OBJECTIVE**

The stability of pool boiling systems is investigated in terms of the model introduced in [5]. Its non-dimensional formulation and steady state solutions are recapitulated hereafter. Furthermore, the control strategy to stabilize the unstable steady states is discussed.

**Pool boiling model description**

The heat transfer within the two-dimensional rectangular heater \( D := [0, 1] \times [0, D] \) (Figure 1) is considered. Its boundary segments comprise (i) \( \Gamma_A : x = 0, 1 \): adiabatic sidewalls, (ii) \( \Gamma_H : y = 0 \): constant heat supply extended with the system input, by which unstable states must be stabilized, and (iii) \( \Gamma_F : y = D \): nonlinear heat extraction by the boiling process. The heat transfer within \( D \) is modeled by

\[
\frac{\partial T}{\partial t} = \kappa \nabla^2 T, \quad \frac{\partial T}{\partial \nu} \bigg|_{\Gamma} = g(x, T) \quad \text{on} \ \Gamma
\]

where \( \nu \) is the outward normal on \( \Gamma \) and where the function \( g \) is defined by

\[
g(x, T) = \begin{cases} 
0, & \text{on } \Gamma_A \\
\frac{1}{\kappa} (1 + u(t)), & \text{on } \Gamma_H \\
-\frac{1}{\kappa} q_F(T_F), & \text{on } \Gamma_F
\end{cases}
\]

with \( T(x, t) \) the non-dimensional temperature excess (i.e. the temperature relative to the boiling point of the medium) and \( T_F(x, t) := T(x, D, t) \) its distribution at the fluid-heater interface \( y = D \). The non-dimensional system parameters are \( \Lambda \) and \( \kappa \) the nondimensional heater conductivity and diffusivity, respectively, \( D \) the heater aspect ratio and \( \Pi_2 \) the ratio between CHF and constant heat supply, see [5]. Since, physical considerations imply \( \Lambda D / \kappa = 1 - \Pi_2 / [5] \), \( \Lambda \), \( D \) (heater properties) and \( \Pi_2 \) (heating conditions) are left as remaining system parameters.

**Two-dimensional rectangular heater.**

The nonlinear heat-flux function \( q_F(T_F) \) describes the local heat exchange between the heater and the boiling fluid. On physical grounds, it is identified with the so-called boiling curve, that is, the relation describing the mean heat exchange between heater and fluid along the entire fluid-heater interface [5]. This implies a functional relation \( q_F(T_F) \) according to Figure 2. Function \( q_F \), equivalent to the boiling curve, comprises three distinct regimes that correspond to one of the local boiling modes and associated mesoscopic boiling states: nucleate boiling (left of local maximum; fluid-rich state); transition boiling (in between both extremes; transitional state); film boiling (right of local minimum; vapor-rich state). Heater surface enhancements to increase pool boiling heat transfer, can be implemented in this model by adapting this local boiling curve.

**Steady states and their stability properties**

Principal topic of interest here is designing a control strategy to stabilize unstable steady states of \( 1 \). Steady state and stability behavior has been considered extensively in [5,6]. 2D steady states can be classified into two groups: \( x \)-independent (“homogeneous”) and \( x \)-dependent (“heterogeneous”) steady states. The present study is restricted to

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1. Here “passive” means the boiling process is uncontrolled.
2. Here mesoscopic means locally averaged in space and time over intervals larger than bubble dimensions and bubble lifetimes so as to smooth out microscopic short-term fluctuations [11].
homogeneous temperature profiles and, consequently, the system reduces to a 1D problem in y. Homogeneous steady state solutions $T_\infty(y)$ to (1) are of the form

$$T_\infty(y) = \frac{D}{\Lambda} \left( 1 - \frac{y}{D} \right) + T_{F,\infty}$$

where $T_{F,\infty} := T_\infty(D)$ is the fluid-heater interface temperature governed by the nonlinear boundary condition $q_f(T_{F,\infty}) = \Pi_2^{-1}$ [5]. Thus $T_{F,\infty}$ coincides with the intersection(s) between the boiling curve (solid line in Figure 2) and the normalized heat-supply $\Pi_2^{-1}$ (dashed line in Figure 2). This implies that 3 homogeneous equilibria exist for the values of $\Pi$ considered, 1 < $\Pi_2$ < $\Pi_1$ ($\Pi_1^{-1}$ equals the local minimum in $q_F$) [6].

The stability analysis in [6] revealed that the open-loop system admits only two stable steady states: homogeneous nucleate ($T_{F,\infty,1}$) and homogeneous film ($T_{F,\infty,3}$) boiling. The homogeneous transition-boiling state $T_{F,\infty,2}$ (and all heterogeneous states) are inherently unstable.

![Figure 2: Local boiling curve (solid line) and constant heat supply (dashed line).](image)

**Stabilization of unstable steady states**

The simplest control strategy is a P-controller $u(t) = K \left( T_{F,\text{set}} - T_F(t) \right)$, regulating $u$ proportionally to the departure of the current interface temperature $T_F$ from a set value $T_{F,\text{set}}$, with $K$ the controller gain. ($T_{F,\text{set}} = T_{F,\infty,2}$ when stabilizing the unstable steady state.) This strategy has been pursued in [8] and analysis of the resultant closed-loop system revealed that stabilization of the system is only possible in specific regimes of parameter space, as is mentioned in the introduction as well. In other words, a P-controller, irrespective of the gain $K$, cannot always stabilize the system.

In order to obtain better overall performance the latter is generalized to the linear state feedback controller

$$u = -K (v - v_{\text{ref}})$$

with $K$ the feedback gain vector and $v$ a vector that represents the temperature distribution within the heater. This controller is compared to the P-controller in [4] and offers a much higher degree of control.

Since in actual pool boiling systems only the temperature at the fluid-heater interface can be measured, an observer (or “state estimator”) must be included, however, that is beyond the scope of the present paper. The interested reader is referred to [4] for details on, how to design a stable observer. Principal objective of the present study is designing the gain vector $K$ such that user-defined properties of the closed-loop system are obtained.

**STATE-SPACE FORM OF THE 1D SYSTEM**

The 1D system (1) is recast into the generic state-space form, see e.g. [12], so as to facilitate analysis of it, using standard control theory. This can be accomplished through spatial discretisation of the 1D model by the Chebyshev tau method [13]. This hinges on the Chebyshev expansion of the temperature profile following

$$T(y,t) = \sum_{p=0}^{P} \tilde{T}_p(t) \phi_p(\theta(y))$$

with $\phi_p(\theta) = \cos(p \arccos(\theta))$ the $p$-th Chebyshev polynomial, $\theta \in [-1,1]$ the computational domain (relating to the physical domain via $\theta = 2y/D - 1$) and $T(t) = [\tilde{T}_0, \ldots, \tilde{T}_P]^T$ the time-dependent Chebyshev spectrum of $T(y,t)$, which can be evaluated via the discrete Chebyshev transformation [13]. Application of the Chebyshev method to (1) yields

$$\dot{T} = \frac{4K}{D^2} D_2 T$$

as ODE for the Chebyshev spectrum, with $D_2$ the discrete Laplace operator in the computational domain [13]. Similarly, discretisation of the corresponding boundary conditions leads to

$$\frac{2\Lambda}{D} \left[ \begin{array}{cccc} 0 & -1 & \cdots & (-1)^P p^2 & \cdots & (-1)^P p^2 \\ 0 & 1 & \cdots & p^2 & \cdots & p^2 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 + u(t) & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\Pi_2 q_F \left( T_F(T) \right) & \cdots & \cdots & \cdots & \cdots & \cdots \end{array} \right] T =$$

with $T_F = \sum_{p=0}^{P} \tilde{T}_p$ the temperature at the fluid-heater interface.

Implementation of the boundary conditions by the tau-method hinges on separation of the spectrum into a low-frequency and high-frequency regime via $T_L = [\tilde{T}_0 \ldots \tilde{T}_{P-2}]^T$ and $T_H = [\tilde{T}_{P-1} \tilde{T}_P]^T$. The tau method consists of dropping the high-frequency contribution in favor of the boundary conditions, which can be done under the proviso of spectral convergence. This reduces the Chebyshev tau discretisation to

$$\dot{T}_L = A_{nl} T_L + B_{nl} v(u, T_L)$$

constituting a nonlinear system of order $n = P - 1$ for $T_L$, with $A_{nl}$ and $B_{nl}$ constant system matrices representing action of Laplace and boundary conditions and $v(u, T_L)$ the nonlinear RHS of the boundary conditions (7). The subscript “L” is dropped hereafter for brevity, see [4].
The homogeneous steady states (3) satisfy the nonlinear system (8) for $u = 0$ and can therefore be analyzed by means of the linearisation of this system around the steady state $T_\infty$ in question. Linearisation of (8) leads to

$$\dot{T} = A_t \dot{T} + B_t u$$

(9)
as linear ODE for the departure $\dot{T} = T - T_\infty$ from the steady state, with

$$A_t = A_{nl} + B_{nl} \begin{bmatrix} 0 \\ -\Pi_2 \gamma \end{bmatrix} C, \quad B_t = B_{nl} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(10)

and $\gamma = \left. \frac{\partial \Phi}{\partial \tau} \right|_{\tau = \tau_F = T_F, \infty}$. Combination with the output $T_F$ as defined before yields

$$\dot{T} = A_t \dot{T} + B_t u$$

(11)

$$\dot{T}_F = CT$$

(12)
as the state-space form of the 1D pool boiling system. Since, $\Lambda$ and $D$ are not individually present in the obtained system and appear only as the ratio $\Lambda/D$, the corresponding system parameters are the ratio $\Lambda/D$ (heater properties) and $\Pi_2$ (heating conditions).

**CONTROLLER DESIGN**

In this section the system is analyzed and the feedback gain vector of the proposed control strategy is designed.

**Controllability**

The controllability of the system (11) can be determined by means of the Hautus test [12]. This test states that (11) is controllable if and only if the matrix $\mathcal{C}(\lambda) = [A_t - \lambda I \ B_t]$ is of full rank for all $\lambda \in \mathbb{C}$. Application of the Hautus test to the system given by (11) reveals, the system is controllable – and, inherently, stabilizable – in almost the entire parameter space. This property is lost only for $\Lambda/D \lesssim 0.1$.

**Control strategy**

The controllability property of the system has the important implication that indeed a state feedback controller of the form (11) can be designed for the pool boiling system. Here this controller takes the form

$$u = -K \dot{T}$$

(13)

**Design of the generic control law**

The properties of the closed-loop system may greatly vary with the choice for $K$ and, consequently, setting the proper gain vector $K$ is imperative for attaining an adequate controller. Generally, the gain vector can be tailored to a specific situation by means of pole-placement algorithms, which for given $A_t$ and $B_t$ determine $K$ such that the controller places all poles at predefined positions. However, the size of the present system precludes employment of conventional pole-placement algorithms. The order of the system – and thus the number of degrees of freedom in the gain vector $K$ – namely is $P \gtrsim 40$ so as to attain spectral convergence of the Chebyshev approximation. Therefore, the feedback gain vector must be designed using an alternative methodology.

For the present system, an appropriate gain vector is determined by means of the Linear-Quadratic-Regulator (LQR) approach [14]. This approach is somewhat looser than pole-placement algorithms in that it seeks to find a possible rather than tailor-made $K$ for a given control task by minimizing the cost function

$$J = \int_0^\infty \left[ T(t)^T QT(t) + u(t)^T Ru(t) \right] dt$$

(14)

with $Q$ and $R$ weight matrices. Depending on the latter, different (yet comparable) gains $K$ may be found. Here $Q$ is identified with the unit matrix, and $R$ is a scalar and is set to unity, i.e. $R = 1$. Figure 3 shows the magnitude of the elements of the gain vector $K$ found via the LQR algorithm for regulation of the unstable steady state and reveals that it exhibits similar exponential decay as the Chebyshev spectrum, i.e. the state vector [13]. This convergence is observed for all values of $\Lambda/D$ and $\Pi_2$. However, the decay rate decreases with decreasing $\Lambda/D$, as is shown in Figure 3(a). The decay rate also varies if $\Pi_2$ is varied, as is shown in Figure 3(b). The decay rate increases as $\Pi_2$ is chosen closer to its upper and lower bound, given by 1 and $\Pi_1 = 4$, respectively. The state weight matrix $Q$ and the input weight matrix $R$ may be chosen to be any matrix in the LQR algorithm. The form of the state weight matrix can be chosen such that only the first or last state component is weighted. Another possibility is to weigh states in a decaying or ascending order. These possibilities all have been pursued for different magnitude of $Q$, as only the magnitude of $R$ relative to that of $Q$ is important, varying $R$ is not considered. It turns out that the form and magnitude of $Q$ does not influence the decay rate of the controller gain vector. Hence, each gain vector, irrespective of the system parameters or weight matrices, exhibits this exponential decay.

This has the important implication that mainly the lowest contributions to the gain vector are relevant for the performance of the controller, which is entirely consistent with the fact that mainly the lowest Chebyshev modes are relevant for the dynamics of the system. Hence, gain vectors are restricted to the form

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & 0 & \cdots & 0 \end{bmatrix}$$

(15)

meaning that the controller calculates the input on the basis of the three lowest Chebyshev modes of the system temperature only. Drawback of this approach is that poles can no longer be regulated unconditionally, as this namely requires all gain components be non-zero. However, this is but a minor limitation. Principal advantage of gain vectors according to (15) is that it facilitates the design of a compact and robust controller.
In Figure 4 the pole trajectory is shown for changing system parameters, see [4]. An interesting observation is that the gain vector element for which stability is gained or lost can differ in these cases. Meaning that the design approach with further increased $k_1$ robust to system parameter uncertainties, i.e. the system is also stable if slightly different parameter values are considered as these might not be known precise enough.

Since $k_1$ can not stabilize the system for $k_2 = k_3 = 0$, the second gain for sure will not be able to stabilize the system for $k_1 \leq 0$, as the first pole can be moved further (in)to the left half plane by the first gain vector element than by the second. Hence, $k_1 = 15$ is chosen in order to move the first pole closer to the imaginary axis. The accompanying pole trajectory plot is given in Figure 5. As can be seen for increasing $k_2$ from $k_2 = 0$, the first pole crosses the imaginary axis and stability is attained. However, if $k_2$ is increased further, the first and second pole coincide and form a complex conjugate pair. Subsequently, the real value of the poles increases and a super critical Hopf bifurcation occurs, through which stability is lost. As a result, $k_2$ must be chosen in between bounds in order to obtain a stable closed-loop system. For increasing/decreasing $k_1$ the global shape of this pole plot stays the same and the entire trajectory moves further into the left/right half plane. This widens/narrows the stability interval for $k_2$.

Finally, the pole plot for $k_3$ is discussed. For increasing $k_2$ from $k_2 = 0$, this pole plot changes drastically. For small $k_2$, the system can not be stabilized by altering $k_3$. For medium values of $k_2$, the second and third pole coincide to form a complex conjugate pair and pass the imaginary axis at the lower bound for $k_3$. As can be seen, in Figure 6 which shows the pole trajectory of $k_3$, with fixed $k_1 = 15$ and $k_2 = 3$. As can be seen, in this case $k_3$ is subject only to a lower bound for stability. On the other hand for larger $k_2$, the first and second pole form a complex conjugate pair and pass the imaginary axis at the lower bound for $k_3$. This is shown in Figure 7 where the pole plot is shown for fixed $k_1 = 15$ and $k_2 = 5$. Here increasing $k_3$ moves the first and second pole towards instability. Decreasing $k_3$ on the other hand first moves the poles further into the left half plane, but eventually towards the right half plane, which means $k_3$
is subject to a lower and upper bound for these values of $k_2$. Hence, optimizing $k_2$ and $k_3$ is very sensitive to parameter uncertainties, meaning $k_3$ must be chosen quite carefully. Therefore, these gain vectors should be kept small in order to keep the controller as robust as possible.

\[ K = \begin{bmatrix} 30 & 10 & -6.6 & 0 & \cdots & 0 \end{bmatrix}, \quad (16) \]
\[ K = \begin{bmatrix} 10 & 2 & -3 & 0 & \cdots & 0 \end{bmatrix}, \quad (17) \]
\[ K = \begin{bmatrix} 50 & 25 & 10 & 0 & \cdots & 0 \end{bmatrix}, \quad (18) \]

respectively. In Figure 8 the first four poles of the three closed-loop systems are given. In the next section the performance of these controllers is tested by means of simulations.

**Figure 6:** pole trajectory plot of varying $k_3$, while $k_1 = 15$ and $k_2 = 2$.

**Figure 7:** pole trajectory plot of varying $k_3$, while $k_1 = 15$ and $k_2 = 5$.

**Figure 8:** closed-loop poles for the system with $\Lambda/D = 1$ and $\Pi_2 = 2$, stabilized by the three different feedback gain vectors $K$.

### Design of user-defined controllers

As can be concluded from the above analysis, the dominant poles of the closed-loop system can be placed at desired locations. This is done with a feedback law with relatively few degrees of freedom. As a result, satisfactory closed-loop dynamics can be obtained by application of a relatively simple control law. The accompanying values for $k_1$, $k_2$ and $k_3$ can be found using pole trajectory plots. The main conclusions drawn from these plots are:

- The only unstable open-loop pole, can be shifted into the left half plane. This pole can be shifted to the left the most by the gain $k_1$ and the least by $k_3$.
- The first and second pole can form a complex conjugate pair. This will result in a sub critical damped system and can be achieved by altering $k_2$ or $k_3$.
- The controller can be made more robust by increasing $k_1$. Increasing $k_2$ and/or $k_3$ might have the opposite effect, as these gains have a lower and upper bound.

Using the pole trajectory plots given in the previous section, feedback gain vectors for the case $\Lambda/D = 1$ and $\Pi_2 = 2$ using a Chebyshev expansion for $P = 60$, have been designed. Desired dynamics exhibit fast, slow and oscillating behavior, the corresponding gain vectors are given by

**CONTROLLED POOL BOILING**

The performance of the state feedback controllers introduced in the previous section is investigated hereafter. The evolution of the (non)linear closed-loop system has been simulated so as to (i) verify the asymptotic stability of the nonlinear system, (ii) establish whether indeed stabilization of unstable states is accomplished and (iii) to obtain an approximation of the region of attraction, i.e. the region in which initial conditions are regulated to the equilibrium. Simulations have been performed using the CONTROL SYSTEMS Toolbox of Matlab. The perturbed transition-boiling state serves as initial condition for the simulations; the perturbation consists of a super imposed offset $\Delta T = \psi n$, with $n$ an exponentially-decaying noise vector with $n_0 = 1$ and $\psi$ an amplification factor.

Simulations have been performed for “weak” ($\psi = 10^{-1.5} \approx 0.03$) and “strong” ($\psi = 1$) perturbations of the unstable steady state, resulting in small and large initial offsets, respectively. The three different closed-loop systems are simulated using the same initial condition, meaning deviations between the evolution of the output $T_F$ of these simulations, solely originate from the difference in feedback law. Figure 9 shows the evolution of the departure $T_F = T_F - T_{F,\infty}$ from the steady state transition-boiling temperature $T_{F,\infty}$ at the fluid-heater interface in case of the weak perturbation $\psi = 10^{-1.5}$, revealing that the linear and nonlinear evolution closely shadow each other and both converge asymptotically onto $T = 0$ for all controllers. This has two important ramifications. First, the nonlinear closed-loop system is indeed asymptotically stable in
proximity of the steady state. Second, the control strategy is capable of stabilizing the unstable transition-boiling state. As can be seen the three feedback laws result in very different behavior. The aimed for fast, slow and oscillating closed-loop behavior is acquired. The nonlinear system is described accurately by the linearized system, for this small deviation from the equilibrium. The slow controller is not able to correspond actively to the typical 'non-minimum phase' behavior of the system, i.e. the output diverges away from its equilibrium before it is regulated towards it, which results in a larger deviation. As a result, the linear and nonlinear system do not behave the same anymore. The nonlinear system, still is stabilized by the linear controller, however.

Figure 9: Evolution of the linear and nonlinear closed-loop systems in terms of the departure $T_F = T_F - T_{F,\infty,2}$ from the steady state transition boiling temperature $T_{F,\infty,2}$ at the fluid-heater interface. Initial condition is the unstable equilibrium subject to a "weak" ($\psi = 10^{-1.5}$) perturbation.

Figure 10 displays the evolution of $T_F$ for the strong perturbation $\psi = 1$. Here the linear and nonlinear evolution are markedly different, meaning that the linear approximation is no longer representative for the nonlinear dynamics. The nonlinear evolution nonetheless still converges onto the asymptotic state $T = 0$, for two of the three controllers. The slow controller is not able to direct the nonlinear system to its equilibrium. Meaning the initial offset is chosen outside the region of attraction of this closed-loop system. Although in the case of the fast and oscillating controller, the linear system does not describe the nonlinear system accurately, the nonlinear system still gets stabilized, signifying retention of asymptotic stability even for large perturbations. This is corroborated by further simulations of the fast system with large($r$) $\psi$, strongly suggesting that the nonlinear closed-loop system exhibits global asymptotic stability.

In Figure 11 the evolution of the input for the three nonlinear systems is given in case of the strong initial perturbation. As can be seen the main advantage of the slow controller is that its input values are drastically smaller than the values of the other two systems. In practical applications this can be of crucial importance, since in practical applications input values are restricted by the limitations of the actuator. Since, the actuator for example can not supply infinite amounts of heat.

Figure 11: Evolution of the input of the nonlinear system. Initial condition is the unstable equilibrium subject to "strong" ($\psi = 1$) perturbation.

CONCLUSIONS
In this study a 2D nonlinear heat-transfer model for pool boiling systems is considered. The model involves only the temperature distribution within the heater and models the heat exchange with the boiling medium via a nonlinear boundary condition imposed at the fluid-heater interface. This compact model is employed for the design and analysis
of a robust control strategy for the stabilization of unstable boiling states. To this end a state feedback controller is implemented that regulates the heat supply as a function of the internal state of the system. The present study is restricted to homogeneous temperature distributions on the fluid-heater interface, reducing the original 2D problem into a 1D problem. This 1D pool boiling system is recast into a generic linear state-space form through spatial discretisation with the Chebyshev tau method so as to facilitate analysis by standard techniques from control theory. The 1D system has one unstable boiling state, physically corresponding with transition boiling, the stabilization of which requires the system to be controllable. Analysis reveals that the system is controllable in most of the parameter space. Hence, regulation of the system is possible for wide parameter ranges of practical interest.

Investigations disclose that the system dynamics are determined predominantly by its lowest Chebyshev modes, meaning that mainly the gain components associated with these modes are relevant for the closed-loop dynamics. Thus state feedback controllers with only three non-zero gain components are considered in the present study. Using pole trajectory plots, user defined properties for the closed-loop system can be obtained by appropriate choice of the three gain vector elements. Since, these gain vectors are designed by means of the linearized system, their performance for stabilization of the nonlinear system requires investigation, in order to establish its value for practical purposes. Simulations of the evolution of the nonlinear system stabilized by different types of controllers, reveal closed-loop behavior is dependent on the type of controller. For a fast controller, the region for initial conditions in which the controller can still stabilize the system is much larger than for a slow controller. The price to pay for this enlarged region of attraction are larger input values, meaning a larger power is required of the actuator. In practical applications the input is bounded by the limitations of this actuator, meaning it might be impossible to use the fast controller. The third controller, which results in a sub critical damped system, also results in asymptotic stability of the nonlinear system. Furthermore, it shows large system input values and a large region of attraction.

These findings imply that the control loop, if designed correctly, indeed is capable of robustly stabilizing the pool boiling system. Moreover, simulations with the fast controller strongly suggest that the nonlinear system exhibits global asymptotic stability.

First exploratory studies reveal that the control strategy proposed here in principle admits extension to 2D (and 3D) systems. This is currently in progress. Moreover, future studies for experimental validation of the performance of control loops are planned.

REFERENCES


