Uncertainty modeling and robust control of Linear Tape-Open drives

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1 Abstract

The Linear Tape-Open technology was developed at the end of the 1990's as an alternative to the commercially available magnetic tape data storage systems. At the moment, it is the most volume-effective way of data storage. In order to keep the technology competitive, the amount of data that can be stored on a single cartridge is getting increased continuously. Due to the higher data density on the tape, the error tolerances for the positioning system of the read/write heads get tighter. To keep up with these demands, an accurate model of the system uncertainties is essential.

In this thesis an uncertainty description that uses the Dual-Youla factorization and an additive uncertainty description are derived and the resulting sets of models are compared. The model sets are based on closed loop frequency response measurements that are performed on two different LTO-3 tape drive setups. During the measurements we discovered that the positioning mechanism suffers from saturation. Another important finding is that the bandwidth of the controlled system is being limited by resonance peaks that appear in the frequency range from 500 to 2000Hz. These peaks cause considerable phase loss and should therefore be captured in the uncertainty model. It was chosen to model the uncertainties using unstructured uncertainty descriptions. The theory behind the Dual-Youla description and the additive uncertainty description is described and conditions for robust stability and robust performance are derived. The results show that there exists a big conservatism difference. This is due to the fact that the Dual-Youla factorization uses knowledge about the controller to find the model set. Finally, the Dual-Youla description is used to synthesize a controller that increases the performance of the positioning system considerably. During the controller design process, the focus is kept on rejecting low-frequent disturbances because previous research has shown that these disturbances are the main cause for positioning errors. The newly designed controller achieves a nominal bandwidth of 700Hz while maintaining good disturbance rejection.
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Linear Tape-Open is a magnetic tape data storage technology that was developed at the end of the 1990’s by the LTO Consortium, a joint-venture by Seagate Hewlett-Packard and IBM, as an alternative to the other magnetic tape data storage systems available. Due to the layout of the cartridges, it is the most volume-effective method to store data on tapes at the moment. Another advantage of this technology is that stored data can be preserved for a relative long time, which makes it ideal for archiving purposes. To keep the LTO-technology competitive with other data storage systems, the amount of data that can be stored on a single cartridge has to be increased continuously. This can be done by adding data tracks to the tape. However, the width of the tape has to stay the same. Therefore, the tracks have to be arranged closer to each other, which results in higher track densities. This demands that the read/write heads have to be positioned with a higher accuracy. Various solutions with respect to tape transport, more enhanced actuators and higher quality tape media have been proposed. However, this report focuses only on one particular aspect: improving the servo performance by characterizing uncertainty in the servo system and providing a robust control algorithm for high track density recording.

Previous research projects on LTO-tape drives focused on the identification of the dynamics and control of a single tape drive (ten Dam 2007). Furthermore, the main sources of disturbances affecting the positioning of the read/write heads were identified. This resulted in the design of high-bandwidth controllers that work on that particular tape drive. However, it could not be guaranteed that the designed controllers would also work on other LTO drives.

This thesis focuses on the design of a high performance controller for multiple LTO-tape drives. First, the frequency responses of two LTO-3 tape drives were measured. The measurement data is used as a basis for two different control design techniques. High-bandwidth PID controllers are designed for both drives, and it is investigated whether a controller designed for one tape drive also works on another. The other technique uses a robust control approach in order to increase performance of a set of models rather than only two separate models. A description of the possible amount of variation in the dynamics is required for this design. The variation in the plants can be expressed using several uncertainty descriptions, e.g. an additive uncertainty description or an
uncertainty description using the Dual-Youla factorization. Both techniques are used here, and the results are compared. For the design of a new robust controller, the least conservative set of models is used as a basis for the design process. Conditions for robust stability and robust performance are derived, and the final controller is optimized to fulfill these conditions.
3 System description

3.1 General info about LTO-3 tape drives

Despite the advances in the technologies of optical and hard-drive storage media, magnetic tape data storage still remains the most cost-effective and therefore a very common way of data storage. To ensure the competitiveness of this technology companies and their research teams are continuously trying to increase the data storage capacity. This is done by using more tracks to store data while the width of the tape is kept constant. Therefore, the accuracy of the read/write head’s positioning system has to be improved accordingly. The LTO data storage system was developed in the late 1990’s by a consortium formed by Seagate, Hewlett-Packard and IBM. It uses single reel magnetic tape cartridges to store data. The first generation of LTO systems, the so-called LTO-1 systems, had a capacity of 100 GB while the newest generation, the LTO-5, can store up to 1.5 TB. This thesis focuses on an intermediate generation, the LTO-3 drive, which has a capacity of 400 GB. A LTO3-tape has a width of half an inch, and consists of 4 data bands with 5 servo bands that enclose each data band, as indicated in Figure 3.1.

![Figure 3.1: LTO tape](image)

Every data band has a width of 190 µm and is split up into 176 data tracks on which the actual data is saved. This corresponds to about 1.08 µm per data track. The read/write head can access 16 tracks at the same time, which means eleven sweeps for a whole data band. The servo bands are used as reference for the positioning of the read-write heads. The figure below shows a schematic representation of such a servo band.

A sensor measures the two distances A and B. The ratio of these distances is then
used as an indicator for the vertical position of the read-write head. This ratio can vary in between 0.3 and 0.7. In the experiments in this thesis, the middle of the servo band was chosen as reference, because the fluctuation of the PES signal is unknown. This reference corresponds to a PES ratio of 0.5058.

A feature of the read-write head architecture is that the read heads are positioned behind the write heads. Because of that data that was written to the tape can directly be checked for errors, and be rewritten in case it was not written properly.

3.2 Experimental facility

The experimental facilities of the LTO drives located at the System Identification and Control Laboratory at UCSD have three ports through which signals can be injected or measured. The input port can be used to add the so-called REF-signal to the system. This signal acts as an input disturbance on the plant. In addition to the REF-signal, the Isense-signal and the PES-signal can be measured. The Isense-signal gives the current that is injected into the plant. This is done by measuring the difference in voltage over the current amplifier that is situated in front of the plant. This voltage signal is then fed through a shunt resistor, which is chosen in such a way that the magnitude of the resulting voltage equals the magnitude of the current sent to the plant. To protect the actuator, the signals amplitude should not exceed 5V. The PES-signal gives an indication of the position error of the read/write head. The difference in time-ratios is converted to a voltage signal by a 10bit DAC-converter. This signal is shifted by +1.25V and it is limited by 0V and 2.5V. The actual position error in micrometer can be calculated using the following formula:

\[
PES_{\mu m} = (\text{measured}_{\text{ratio}} - \text{target}_{\text{ratio}}) \times 475.718
\]  

(3.1)
The values for parameters $min_{ratio}$, $max_{ratio}$ and $target_{ratio}$ can be programmed using the firmware of the tape drive by the user.

A so called "Seagate serial diagnostic adapter" is attached to the experimental facility. This adapter is controlled by a PC and gives the user the possibility to send commands to the firmware. There are two different types of commands that can be used, servo commands and controller commands. The servo commands are used to experimental facility the internal controller and the analog-to-digital converter that produces the position error signal. One can specify the maximum, the minimum, and the target timing ratio to make optimal use of the 10bit resolution of the converter. It is also possible to deactivate the internal controller, which can be useful if one wants to implement an external controller using the input and output ports of the experimental facility. This option is also useful to measure the disturbances resulting from the lateral tape motion, which can be significant when designing a controller as older research indicates. The controller commands are used to load and unload the cartridges and to control the speed at which the tape is unwound. To simulate normal working conditions, a tape speed of 4 m/s was chosen. At this speed, it takes 2 minutes and 47 seconds to fully unwind one cartridge. This information can be useful for the calculation of the number of consecutive frequency response measurements that can be performed after each other.

A spectrum analyzer was used to create the excitation signals, and to measure and interpret the output signals. The analyzer contained a signal generator, which can produce different signals like harmonic sinusoids, random noise or colored noise. These signals are sent through REF-port to the actuator, as will be described in the chapter "Closed loop measurements". Another useful feature of the spectrum analyzer is the ability to calculate frequency responses while the measurements are still going on. Besides saving time this also reduces the amount of data that has to be transferred to the PC, because the time-domain data can be discarded and only the frequency-domain has to be kept.

4 System identification

4.1 Measured signals

In order to design a controller for the vertical actuator of the read/write heads, one must first have a model of the dynamics of this actuator. This can be done by deriving
a theoretical model, which is a rather complicated and time consuming task. A more realistic approach is to create a model by measuring the frequency responses of the LTO-drives and this allows direct verification of models against experimental data. This is done by using the input and output ports on the experimental facilities. A block diagram of the tape-drive’s control scheme is shown in the figure below. It can be seen that there are two different control loops, an internal and an external control loop. During normal operation only the internal loop is used. The external loop can be used by an engineer to implement a new controller without having to reprogram the firmware. A disadvantage of the external control loop is that it contains three more delays than the internal loop, which limits the controller performance that can be achieved.

![Block diagram of the system](image)

Figure 4.1: Block diagram of the system

In this block diagram the actuator, the internal controller and the external controller are respectively denoted by $P(j\omega)$, $C_i(j\omega)$, $C_e(j\omega)$. Furthermore the analog-to-digital converters are denoted by (ADC) and the digital-to-analog converters, which also introduce a delay of half the sampling time, are denoted by (DAC). By injecting a disturbance signal into the REF-port two frequency responses can be measured. Those responses are given by:

$$REF \rightarrow PES : PS(j\omega) = \frac{\alpha \beta \mu e^{-j\omega(\tau_{REF} + \Delta T + \tau_{PES})} P(j\omega)}{1 + \beta K_{loop} e^{-j\omega(\tau_{REF} \Delta T/2)} C_i(j\omega) P(j\omega)}$$ (4.1)

$$REF \rightarrow I_{sense} : S(j\omega) = \frac{\alpha \beta \gamma e^{-j\omega(\tau_{REF} + \Delta T/2)}}{1 + \beta K_{loop} e^{-j\omega(\tau_{REF} \Delta T/2)} C_i(j\omega) P(j\omega)}$$ (4.2)
where,
\[
\begin{align*}
\alpha, K_{\text{loop}} &= \text{programmable gains} \\
\beta &= \text{amplifier gain} \\
\gamma &= \text{amplification of } I_{\text{sense}} \\
\mu &= \text{programmable gain in PES} \\
\tau_{\text{REF}} &= \text{delay in REF-signal (} \approx \Delta T) \\
\Delta T &= \text{sample time (} = 5 \times 10^{-5} \text{s}) \\
\tau_{PES} &= \text{delay in PES computation}
\end{align*}
\]

By dividing the two responses a model for the actuator can be derived:
\[
P(j\omega) = \frac{\gamma e^{j\omega(\tau_{PES}+\Delta T/2)}}{\mu} PS(j\omega) S(j\omega) \tag{4.3}
\]

This expression gives a description of the actual plant, but as can be seen from the block diagram above a controller also has to take into account the dynamics of other blocks in the control loops. Therefore two different plants are defined, an internal plant for the internal control loop and an external plant for the external loop respectively.

\[
P_{\text{internal}}(j\omega) = K_{\text{loop}}\beta e^{-j\omega(\Delta T/2)} P(j\omega) = \frac{K_{\text{loop}}\beta e^{j\omega\tau_{PES}} PS(j\omega)}{\mu S(j\omega)} \tag{4.4}
\]

\[
P_{\text{external}}(j\omega) = \alpha\beta e^{-j\omega(\Delta T/2+\Delta T/2+\tau_{\text{REF}}+\tau_{PES})} P(j\omega) = \frac{\alpha\beta e^{-j\omega(\tau_{\text{REF}}+\Delta T)} PS(j\omega)}{S(j\omega)} \tag{4.5}
\]

The digital-to-analog converter in the internal loop was added to the internal and external plants, because the plants are sampled in discrete time.

4.2 Closed loop measurements

The goal of the measurements is to calculate the plant dynamics with a high frequency axis resolution using high coherence process sensitivities and sensitivities measurements in the frequency range of 100-3000 Hz.

Because of the limited power that can be injected into the servo loop via the REF signal, excitation of the servo loop is done over different frequency interval bands to improve signal to noise ratio during identification. For measuring the process sensitivity, a burst chirp signal results in the highest coherence level in all of the intervals. For the
sensitivity measurement however, pink noise for frequencies up to 400 Hz and random noise for higher frequencies turn out to be the best options. During each measurement the noise amplitudes are increased until the maximum voltage level of the PES or REF signal is reached. To collect as much data as possible within one measurement, the measurement is done during the whole time it takes to unwind the tape. Using the maximum resolution of 1600 points per interval, the data is averaged to obtain smooth data sets. The following measurements were done on the first tape drive system with the measureable PES range of $0.5058 \pm 0.0017$, which corresponds to a variation of $\pm 0.8\mu m$ around the target ratio. The Bode plots of the measurements are given in the figures below.

Figure 4.2: Process sensitivity measurement for tape drive 1
The signal to noise ratio is near 1 at the frequency range 100-3000 Hz, so this measurement meets the requirements stated at the beginning of this chapter. Some of the data in different frequency intervals overlap, in section 3.4 this issue will be dealt with using a data filtering algorithm. During the measurements the system showed some non-linear behavior. This is discussed in the following paragraph.
4.3 Non-linear map

When using higher amplitude excitation signals the magnitude of the process sensitivity scales down, which means the system behaves non-linear. This phenomenon is caused by the physical limitation on control signals (Isense) and error checks on the PES in case
of large disturbance on the servo loop. Measurement data obtained with low amplitude excitation signals produces significantly noisier data sets. However, since it does operate in the linear region, the magnitude trend can be used as an indicator for the magnitude of the linear approximation of the system. Measurements done with high amplitude excitation signals have the benefits of smoother data sets and higher coherence values.

To identify the non-linear map, sinusoids at several frequencies with different amplitudes were used to measure the process sensitivity Bode diagram. Since the phase was the same for all amplitude levels, only the magnitude is of interest. The fact that only the magnitude, and not the phase, is influenced shows that the Bode gain-phase relationship does not hold.

![Figure 4.6: Non-linear map in the process sensitivity](image)

The non-linearity is clearly present around 300 Hz, where the disturbance excitation causes the largest fluctuation in the PES and control signal. The non-linear map at 300 Hz that shows the relation between input-and output signal amplitude is shown in Figure 4.7.

In this figure, five measurements are shown using sinusoid inputs at 300 Hz. For every measurement a sinusoid with different amplitude was injected. From the results can be concluded that the gain from REF to PES signal decreases with increasing input signal amplitude. This causes the magnitude of the measured process sensitivity to be lower than expected when injected high amplitude excitation signals. For the sake of simplicity, it will be assumed that the controller output is low and that the system operates in the linear region at all times. Therefore, the magnitude in the measured process sensitivity has to be scaled accordingly. This is done by measuring the process sensitivity again with low amplitude excitation signals to determine the trend of the magnitude. The data that
was measured using high excitation disturbance signals will then be scaled up to match
the data obtained by using low amplitude excitation signals. This way the smooth and
high coherence data of the first measurement can still be used to determine the frequency
response even though the behavior was non-linear. The resulting transformation is shown
in Figure 4.8.

![Non-linear map in the process sensitivity at 300 Hz](image)

Figure 4.7: Non-linear map in the process sensitivity at 300 Hz

The blue line indicates the process sensitivity data that was measured with high
amplitude excitation signals. The green line indicates the measurement data obtained
by the low amplitude excitation signal and the red line indicates the high amplitude
measurement data, mapped onto the trend of the low amplitude data. The mapping
is done by averaging the magnitude of all points within a certain interval for the blue
and the green line, and multiplying every point of the blue line with the quotient of
magnitude between the two. Note that the coherence and phase are unaltered, because

![Scaled process sensitivity data of tape drive 1](image)

Figure 4.8: Scaled process sensitivity data of tape drive 1
the non-linear map only affects the magnitude of the transfer function. For frequencies above 2 kHz no scaling was necessary. The end result is that high excitation level could be used for excitation to improve the signal to noise ratio and the resulting coherence. Phenomena due to static non-linearity are compensated by the calibration proposed in this section.

4.4 Data preprocessing

The frequency responses of the process sensitivity and the sensitivity have been measured in several frequency intervals. Because of that, the FRFs have high resolutions in the frequency domain and the measurement data is rather noisy. Furthermore, because of the overlapping of the intervals multiple measurements might be available at one frequency. Finally, it is also possible that the FRFs of the process sensitivities and sensitivities do not have the exact same frequency grid. For the calculation of the plant smooth data is preferable, and it is necessary that the frequency grids coincide. Therefore, the measurement data has to be preprocessed. The preprocessing is done in two steps. First, the data is checked for frequencies with more than one measurement. If such a frequency points is found, only the response with the best coherence is kept. The other responses are discarded. After that, the frequency space is divided into a certain number of intervals.

To improve the variance properties of the estimated frequency responses, additional data convolution is performed. This is done by averaging neighboring frequency domain points. For the identification of the LTO3-drive it was chosen to split the frequency space in 8000 intervals ranging from 2Hz to 5197Hz. This resolution may seem too fine. However, it was the intention to capture even small resonances since those affect the phase considerably around the expected bandwidth. Before the preprocessing process, the datasets for the process sensitivity consisted of about 11,000 data points, while the datasets for the sensitivity consisted of about 9600 points. After the preprocessing, only 4400 points were left in both datasets. So, ca. 60% of the process sensitivity points and 50% of the sensitivity points were deleted, without affecting the representation of the dynamics. The resulting sensitivities were compensated for delays, such that they can be used as internal loop sensitivities. The Bode diagrams are shown in Figure 4.9.

Both sensitivities have a maximum value of about 10 dB. It can clearly be seen
Figure 4.9: Sensitivity functions compensated for delay

that the resonance peak in the second tape drive lays at a lower frequency than the
resonance of the first drive. Also in the range from 300-3000 Hz a lot of minor differences
can be seen. These differences in dynamics have to be accounted for when designing
a controller. In the phase plot one can see that the delay was compensated correctly,
because the phase approaches zero for high frequencies. The results of the measurements
of the process sensitivities for both plants are shown in Figure 4.10.

Figure 4.10: Process sensitivity functions for both tape drives
The shapes of the process sensitivities are about the same for both tape drives. However there are a lot of minor differences from the frequency range of 300-3000 Hz.

4.5 Open loop frequency data

The Process sensitivity and Sensitivity data now have the same frequency grid. Since the controller that will be designed is supposed to be implemented in the internal loop, the internal plant dynamics are computed as in equation 2.5. The resulting internal plants for both tape drives are shown in Figure 4.11.

![Figure 4.11: Internal plants of both tape drives](image)

Overall the plant dynamics are the same. Both start off with a zero slope in the magnitude, then after the first resonance peak the slopes are minus 40 dB per decade and after the second resonance peak they have a slope of minus 80 dB per decade. However, the plants have resonance peaks at different frequencies and in between 500-2000 Hz there are some minor differences which may lead to unexpected phase loss when not accounting for it. In the low frequent region and high frequent region measurement noise is present. Therefore, the frequency response data at these regions cannot be trusted. However, using simple mechanical insight for a 2-mass system one can conclude that at low frequencies the plants have a constant magnitude and at high frequencies the plants have at least a slope of minus 80 dB per decade, depending on the unknown high frequent dynamics.
5 Control design

The control design of the tape drives aims at finding one controller that minimizes the servo error (PES) while moving the read/write head as fast as possible for both plants. In this chapter first loop shaping is used to design two separate PID controllers. Also a robust control design technique will be used that involves the estimation of a set of models built up from a nominal model equipped with a characterization of the model uncertainty. A comparison will be made between additive uncertainty, which uses only knowledge about the plants, and the Dual-Youla uncertainty model, which also uses controller data that is known beforehand. In order to ensure closed loop performance enhancement while performing the subsequent steps of model set estimation and robust control design, closed loop validation tests are formulated.

5.1 Individual PID control

The computed internal discrete time plant data is used to design two individual discrete PID controllers for both plants. The goal of each PID controller is to have high gain feedback at low with a bandwidth of 600 Hz, a roll-off at high frequencies to suppress high frequent noise and to maintain the following robustness margins:

\[
\max(|S(j\omega)|) \leq 6 \text{ dB} \tag{5.1}
\]

\[
\text{Phase margin} \geq 30 \text{ deg} \tag{5.2}
\]

Here \(\omega_c\) is the crossover frequency of the open loop. The resulting PID controllers are shown in Figure 5.1. They will both be tested on the other plant, which it was not designed for, to see whether the differences in plant dynamics have a large influence on the performance and stability or not.

Both controllers have about the same shape. A notch is used to suppress the first resonance peek in the plants. This causes the sensitivity bump to be lower and thus have a better stability margin. Furthermore a lot of phase lead is added near the bandwidth, which is 600 Hz and a roll off is designed for frequencies over 10 KHz. In Figure 5.2 the open loops and sensitivities for controller K1 on both measured plants are shown.

The maximum sensitivity value for the first controller and plant is 6 dB and the
Figure 5.1: Individual designed PID controllers

Figure 5.2: (L) Open loop functions for controller K1, (R) Sensitivity functions for controller K1

phase margin is 40 degrees which means the control design goals are met. However when the first controller is implemented on the second plant the closed loop is unstable,
because the notch suppressing the first resonance peak is now misplaced. This causes the magnitude of the open loop to go below 1 and have a particularly significant amount of phase loss in that region. In the Nyquist plot this is actually a clockwise encirclement of the -1 point that makes the closed loop system unstable. The second controller, designed for the second plant is also tested on both plants. Again the open loop and sensitivity Bode diagrams are shown in Figure 5.3.

![Bode diagrams](image)

Figure 5.3: (L) Open loop functions for controller K2, (R) Sensitivity functions for controller K2

The misplaced notch at the low frequent resonance peek again causes the magnitude of the open loop to go below 1. However this time there is phase lead, which doesn’t cause the closed loop system to become unstable. Still the performance is a lot worse at low frequencies, because of the misplaced notch. The end result of this exercise indicates that a robust controller needs to be designed that can handle the variations in plant dynamics. This will be discussed in the next chapter.
5.2 Robust control design

In this chapter a robust control design technique will be used to obtain a controller that stabilizes and enhances the performance on both tape drives. This is done by first formulating a relatively simple nominal model and characterize any perturbations of the measured frequency responses compared to the model as uncertainty. The choice of the uncertainty model will prove to be essential for the ability to design a robust controller and the differences will be shown between additive and a control relevant uncertainty description based on co-prime factor perturbations. The approach in this report will closely follow the results, which are shown in (de Callafon 1998). The stability and performance robustness will be characterized by formulating closed loop validation tests. The goal will then be to design a controller that achieves robust performance. Only the resulting definitions and stability and performance robustness conditions will be shown. For full derivation and background on these conditions, one is referred again to de Callafon.

5.2.1 Nominal plant

The two plants that were measured have different resonance peaks. To make sure all resonance peaks in between these two plants are captured in the set of models, the nominal model will have resonance peaks in between the two plants’ peaks. The nominal model has a low order because there was not enough data available to see if the other (little) resonances are common on these tape drives, so those will be captured in the uncertainty model instead. First two low (6th) order fits are computed using a gradient based Gauss-Newton iteration and aims at the minimization of the 2-norm of a weighted difference between the frequency response of the model and the FRF plant data (Levi 1959, Dennis and Schnabel 1983).

The fits both have 2 resonance peaks and go from a zero slope at low frequencies to a minus four slope at high frequencies. This is a common form of a two-mass mechanical servo system. Each of the resonance peaks $H_i(j \omega)$ can be written as:

$$H_i(j \omega) = \frac{1}{(j \omega)^2 + 2\beta_i \omega_{n,i}(j \omega) + \omega_{n,i}^2}$$ (5.3)

The nominal model $P_x$ is designed in such a way that both the natural frequency and the damping coefficient of both plants are averaged for each of the two dominant
\[
\beta_{x,i} = \frac{\beta_{1,i} + \beta_{2,i}}{2}, \quad \omega_{n,x,i} = \frac{\omega_{n,1,i} + \omega_{n,1,i}^2}{2} \text{ for } i = 1, 2
\]  \hspace{1cm} (5.4)

This results in the following nominal model \( P_x \).
As can be seen, the resonance peaks lay in between the original resonance peaks of the plants.

### 5.2.2 Dual-Youla Factorization

The Dual-Youla factorization uses co-prime factorizations for the plant and controller models in order to give an alternative formulation of internal stability of a feedback connection. A smart way to use the Dual-Youla factorization is to use the known data of a stabilizing controller. To construct the uncertainty model or a ”set of models” that is stabilized with one controller and used to design a new controller was done before by (de Callafon 1998). The Dual-Youla factorization describes a system $P_0$ in the following way.

![Diagram of Dual-Youla factorization](image)

**Figure 5.6: Dual-Youla factorization**

In this block diagram the co-prime factorizations of the auxiliary plant $P_X$ and the controller $C$ are used in the following form.

\[ P_x = D_x^{-1}N_x \quad (5.5) \]

\[ C_i = D_c^{-1}N_c \quad (5.6) \]

By choosing the co-primes wisely, the uncertainty description can be influenced in a favorable way. This is however not the scope of this project. The motivation of
using fractional representations for dealing with closed loop data and for closed-loop identification has been addressed in (Hansen 1989, Schrama 1992). In this thesis the Dual-Youla format was not used for the closed loop identification process, but only for the construction of the set of plants. For the sake of simplicity, the co-primes of the plant and the controller were chosen trivially to be the following.

\[
\begin{align*}
N_x &= P_x \quad D_x = I \\
N_c &= C_i \quad D_c = I
\end{align*}
\] (5.7)

If the plant \( P_0 \) is also written in a co-prime factorization, the following formula for \( \Delta_R \) can be derived:

\[
P_0 = D_0^{-1}N_0 = (N_x + D_c\Delta_R)(D_x - N_c\Delta_R)^{-1} = (P_x + \Delta_R)(I - C_i\Delta_R)^{-1} \] (5.8)

\[
\Delta_R = D_c^{-1}(I + P_0C_i)^{-1} (P_0 - P_x) D_x = (I + P_0C_i)^{-1} (P_0 - P_x)
\] (5.9)

### 5.2.3 Uncertainty model

Basically the unknown plant \( P_0 \) can be seen as an uncertain plant with both inverse multiplicative and additive uncertainty for the co-prime choices that were made. The formula for \( \Delta_R \) can be used to characterize the uncertainty \( \|\Delta\|_\infty \leq 1 \) around a nominal model \( P_x \) and using a stable and stably invertible upper bounding weighting filter \( \hat{W}^{-1} \) on \( \Delta_R \), such that:

\[
|\hat{W}(j\omega)^{-1}| \geq \Delta_R(j\omega) \quad \forall \omega
\] (5.10)

\[
\Delta_{unc} := \Delta_R\hat{W}
\] (5.11)

This unknown, but bounded perturbation \( \Delta_{unc} \) takes into account the incomplete knowledge of the plant \( P_0 \), which should lie in the set of models \( P \) that is defined as follows.

**Definition 1** Let a nominal model \( P_x \) with a rcf \((\hat{N},\hat{D})\) and a controller \( C_i \) with a rcf \((N_c,D_c)\) form an internally stable feedback connection \( T(\hat{P},C_i) \). Then the set of models
\( P \) is defined by:
\[
P \left( P_x, C_i, \hat{W} \right) := \left\{ P \mid P = (P_x + \Delta_R)(I - C_i\Delta_R)^{-1} \text{with} \Delta_R \in RH_\infty \text{and} \Delta_{unc} := \Delta_R\hat{W} \text{satisfies} \|\Delta\|_\infty < 1 \right\}
\] (5.12)

To make sure both of the measured plants are captured in this set of models \( P \), the formula for \( \Delta_R \) is used on both plants, such that:
\[
\Delta_{R1}(j\omega) = (I + P_1(j\omega)C_i(j\omega))^{-1}(P_1(j\omega) - P_x(j\omega))
\] (5.13)
\[
\Delta_{R2}(j\omega) = (I + P_2(j\omega)C_i(j\omega))^{-1}(P_2(j\omega) - P_x(j\omega))
\] (5.14)
\[
\Delta_R(j\omega) = \max (\Delta_{R1}(j\omega), \Delta_{R2}(j\omega)) \forall \omega
\] (5.15)

The advantage of the Dual-Youla uncertainty characterization over other uncertainty characterizations, e.g. the additive uncertainty characterization, is that it uses knowledge of the controller to describe the uncertainty. It should be noted that the computation \( \Delta_R(j\omega) \) can be extended to an arbitrary (large) set of measured frequency responses \( P_k(j\omega), k = 1, 2, \ldots, N \) by computing:
\[
\Delta_{R,k} = D_c^{-1}(I + P_kC_i)^{-1}(P_k - P_x)D_x
\] (5.16)
\[
\Delta_R(j\omega) = \max_{k=1,2,\ldots,N} \Delta_{R,k}(j\omega)
\] (5.17)

For the additive uncertainty characterization, the initial controller is chosen to be equal to zero \( (C_i = 0) \). The requirement of \( \Delta_R \) being stable is still met, because all of the plants are stable. In this case, the formula for \( \Delta_{A1} \) is reduced to the following:
\[
\Delta_{R1}(j\omega) = \Delta_{A1} = (P_1(j\omega) - P_x(j\omega))
\] (5.18)
\[
\Delta_{R2}(j\omega) = \Delta_{A2} = (P_2(j\omega) - P_x(j\omega))
\] (5.19)
\[
\Delta_R(j\omega) = \Delta_{A}(j\omega) = \max (\Delta_{R1}(j\omega), \Delta_{R2}(j\omega)) \forall \omega
\] (5.20)

The PID controller \( C_i \) that constructs the set of models \( P_i \) is designed by using simple loop shaping on both plant models. A low bandwidth controller (400 Hz) was preferred because it has to stabilize both the measured plants and the nominal plant model.
Figure 5.7: Initial designed controller $C_i$

With this controller $C_i$, the nominal plant $P_x$ and the uncertainty perturbation $\Delta_R$ can be computed and used as an uncertainty model by upper bounding it with a weighting filter. The following graph shows the uncertainty set for both additive uncertainty and for the Dual-Youla factorization.
In the figure the tight upper bounding function $\hat{W}^{-1}$ can be seen. When tightening the upper bound on the uncertainty around the bandwidth, the performance can be enhanced. However the model becomes less conservative, which means that less possible plants are captured within the set. Note that for low frequencies the choice was made not to upper bound $\Delta_R$, because this part was caused by measurement noise in the plant and there is no actual uncertainty present there.

5.2.4 Stability and performance robustness

The set of plants $\mathcal{P}$ can be written as a lower fractional transform (LFT) framework $Q$. 

![Diagram of LFT](image)

Figure 5.9: Framework $Q$ as an LFT
\[ F_u(Q, \Delta) := Q_{22} + Q_{21} \Delta (I - Q_{11} \Delta)^{-1} Q_{12} \]  
\[ P = \{ P | P = F_u(Q, \Delta) \text{ with } \Delta_{\text{unc}} \in RH_{\infty}, \| \Delta_{\text{unc}} \|_{\infty} < 1 \} \]

\[ Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} = \begin{bmatrix} W^{-1}C_i & W^{-1} \\ (I + P_xC_i) & P_x \end{bmatrix} \]

This framework can again be used to derive a performance characterization that is used in the LFT framework \( M \). This is done by adding the controller \( C \), which has to be tested, in the closed loop within \( M \) and adding performance channels (\( w \) and \( e \)) and performance filters (\( U_1 \) and \( U_2 \)).

![Figure 5.10: Framework M as an LFT](image)

With this framework one can derive the different compartments of \( M \). The performance and uncertainty channels can now be respectively perturbed by the two blocks \( \Delta_P \) and \( \Delta_{\text{unc}} \).

\[ M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \]
Here, the initial controller that was used to derive the set of models is denoted by $C_i$ and the newly designed, currently implemented controller is denoted by $C$. The performance filters are denoted by $U_1$ and $U_2$.

**Structured singular value**

The structured singular value of $M$ can now be computed, which is a matrix function that is very helpful when evaluating the performance robustness (Packard and Doyle 1993). The structure $\Delta$ depends on the structure of the uncertainty set and the performance objective function. The structured singular value with respect to the structure $\Delta$ is denoted by $\mu_{\Delta}(M)$ and is defined by (Packard and Doyle 1993) as follows.

$$
\mu_{\Delta}(M) := \begin{cases} 
\frac{1}{\max_{\Delta \in \Delta} \sigma(\Delta)} & \text{if } \exists \Delta \in \Delta \text{ s.t. } \det(I - M\Delta) = 0 \\
0 & \text{if } \nexists \Delta \in \Delta \text{ s.t. } \det(I - M\Delta) = 0
\end{cases} \quad (5.25)
$$

In this formula $\sigma(\Delta)$ denotes the maximum singular value of $\Delta$. In our case this structure is defined the following.
\[ \Delta := \begin{bmatrix} \Delta_{\text{unc}} & 0 \\ 0 & \Delta_p \end{bmatrix} \bigg| \Delta_{\text{unc}}, \Delta_p \in RH_{\infty}, \|\Delta_{\text{unc}}\|_{\infty} < 1, \|\Delta_p\|_{\infty} < 1 \]  

(5.26)

Where the size of \( \Delta_{\text{unc}} \) and \( \Delta_p \) are compatible with the size of \( M_{11} \) and \( M_{22} \). For the general case the structured singular value will be approximated by a lower and an upper bound on \( \mu_\Delta(M) \). However for the special case of \( \Delta \) and \( M \) used here, the values can be computed exactly. The results of \( \mu_\Delta(M) \) can now be used to study the upper LFT \( F_u(M, \Delta) \) and evaluate both stability and performance robustness by using standard results in literature (Packard and Doyle 1993, Zhou and Glover 1996).

**Stability robustness**

The condition for robust stability reads as follows.

**Lemma 1** Let the stable transfer functions \( M, \Delta \in RH_{\infty} \) construct a basic perturbation model \( F_u(M, \Delta) \) and assume that for all \( \Delta \in RH_{\infty} \) with \( \|\Delta\|_{\infty} < 1 \) the transfer function \( M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \) does not exhibit unstable pole/zero cancelations. Then \( F_u(M, \Delta) \) is well-posed and BIBO-stable for all \( \Delta \in RH_{\infty} \) with \( \|\Delta\|_{\infty} < 1 \), if and only if

\[ \|M_{11}\|_{\infty} < 1 \]  

(5.27)

Combining this result with the fact that the set of models \( P_i \) is stabilized by the controller \( C_i \) results in the following robust stability condition (de Callafon 1998).

**Lemma 2** Consider the set \( P_i \) given in (5.12) and a controller \( C \) such that the feedback connection \( T(P_x, C) \) is well-posed, internally stable and satisfies \( T(P_x, C) \in RH_{\infty} \). Then the feedback connection \( T(P, C) \) is well-posed and internally stable for all \( P \in P_i \) if and only if

\[ \|M_{11}\|_{\infty} = \left\| -\hat{W}_i^{-1}(I + CP_x)^{-1}(C_{i+1} - C_i) \right\|_{\infty} < 1 \]  

(5.28)

An important conclusion can be drawn from this analysis:

If one chooses to keep the new controller \( C_{i+1} \) the same as the controller \( C_i \) used to set
up the set of possible plants, the Dual-Youla factorization guarantees robust stability, since the following equation applies.

\[ \| M_{11} \|_\infty = \left\| -W_i^{-1}(I + C_i P_x)^{-1}(C_i - C_i) \right\|_\infty = 0 \]  

(5.29)

Additive uncertainty description cannot guarantee that, because no controller data is used in the uncertainty description. \( \| M_{11} \|_\infty \) for additive uncertainty is found simply by choosing \( C_i = 0 \), resulting in the following.

\[ \| M_{11} \|_\infty = \left\| -\Delta_R(I + C_i P_x)^{-1}C_i \right\| \]  

(5.30)

To illustrate this, \( \| M_{11} \|_\infty \) has been plotted in Figure 5.12 for both additive and Dual-Youla uncertainty descriptions, using only the initial controller \( C_i \).

![Figure 5.12: Robust stability test for the initial controller \( C_i \)](image)

The blue line in Figure 5.12 shows the additive uncertainty description. Since no upper bound was made for this uncertainty, the actual FRF data of \( \Delta_R \) is used instead of \( \hat{W}^{-1} \). The additive description indicates that the controller \( C_i \) does not stabilize all plants within the additive uncertainty set. However, this controller is known to stabilize all plants within the Dual-Youla set, because \( \| M_{11} \|_\infty = 0 \). This shows that the additive uncertainty description is considerably more conservative than the Dual-Youla uncertainty description. In order to use the additive uncertainty in a less conservative way,
a structural uncertainty approach would be required, because the dominant resonance
peeks can also be represented by a parametric uncertainty.

**Performance robustness**

Performance robustness is a much stronger requirement than stability robustness,
because both perturbation loops in figure 4.11 are now considered.

**Lemma 3** Consider the stable transfer functions $M, \Delta \in RH_\infty$ where $M$ is partitioned
as in equation (5.24) and $\mu_{\Delta}(M)$ is defined to relate to the structure $\Delta$ given in equation
(5.25). Then $F_u(M, \Delta)$ is well-posed, BIBO stable and $\|F_u(M, \Delta)\|_\infty < 1$ for all $\Delta \in RH_\infty$ with $\|\Delta\|_\infty < 1$ if and only if

$$\mu_{\Delta}(M) \leq 1$$  \hspace{1cm} (5.31)

The sensitivity function was chosen as the only indicator for the performance of the
controller. Therefore, the performance weights $U_1$ and $U_2$ were chosen to be:

$$U_1 = \begin{bmatrix} 0 & 0 \\
0 & I \end{bmatrix}; U_2 = \begin{bmatrix} 0 & 0 \\
0 & W_S \end{bmatrix}$$  \hspace{1cm} (5.32)

Where $W_S^{-1}$ indicates the sensitivity weighting filter, which is chosen to have the
following shape.

![Performance weighting filter on the sensitivity function](image)

Figure 5.13: Performance weighting filter on the sensitivity function

This filter will force the controller to have integral action at low frequencies and a
roll-off at high frequencies. The magnitude of the resulting sensitivity function will be
bounded, which forces the controller to have phase lead around the bandwidth. By using these performance filters, $M$ becomes the following.

$$M = \begin{bmatrix} -W_i^{-1} \frac{C-C_i}{1+CP_x} & 0 & W_i^{-1} \frac{1}{1+CP_x} \\ 0 & 0 & 0 \\ \frac{(1+C_iP_x)CW_S}{1+CP_x} & 0 & \frac{W_S}{1+CP_x} \end{bmatrix}$$  \hspace{1cm} (5.33)

To summarize the (robustness) test for a newly designed controller $C = C_{i+1}$, the following tests can be performed, when using the Dual-Youla uncertainty description.

- Check nominal performance of the initial controller by looking at the weighted nominal sensitivity function $W_s(1 + CP_x)^{-1}$.

- Check robust stability of the newly designed controller by checking $\|M_{11}\|_\infty < 1$ for $C = C_{i+1}$, where $M_{11}$ is given in (5.24).

- Check robust performance of the newly designed controller by inserting $C = C_{i+1}$ into $M$ and calculating the structured singular value of $M$ for each frequency and test $\mu_\Delta(M) \leq 1$.

After performing these tests, the structured singular values for the old and the new controller are compared to see if performance has increased. When performance robustness is not achieved, a new controller is designed and all steps are repeated. In case robust performance is not achieved after a number of iterations, one should consider changing the performance filters or tightening the upper bound of the uncertainty mode. Also a different initial controller and/or a different nominal model could be chosen. To start the design of a robust controller the structured singular value for $M$ using the initial controller is given in Figure 5.14.
Figure 5.14: Performance test for initial controller $C_i$

The blue and the green line in Figure 5.14 indicate respectively the structured singular value for the additive and Dual-Youla approach, by using the FRF data of $\Delta_R$. The red line indicates the structured singular value when using the Dual-Youla with upper bound $\hat{W}^{-1}$. When using an additive uncertainty description the structured singular values are a lot higher than with the Dual-Youla factorization. Therefore, it is more difficult to achieve robust performance with the additive description. A new controller $C_{i+1}$ is designed to achieve a higher performance. The nominal bandwidth is 700 Hz. In Figure 5.15 the new controller is shown.
The first step is to test the nominal performance for the controller $C_{i+1}$. The result is shown in Figure 5.16.

The nominal sensitivity function for the newly designed controller $C_{i+1}$ lies under the performance filter. Therefore, nominal performance has been achieved. The next step is the stability robustness test which is shown in Figure 5.17.

Similar as seen for the initial controller $C_i$, an additive uncertainty model does not guarantee robust stability for the new controller $C_{i+1}$, because the low frequent area peaks above 0 dB. Clearly, the additive uncertainty description is too conservative in capturing the perturbation in the measured frequency responses. This is greatly improved by the Dual-Youla based uncertainty model and is caused by the fact that Dual-Youla uses
known controller data in a non-conservative way and the additive description uses no controller data. The structured singular values of $M$ with the new controller are plotted in Figure 5.18.

Figure 5.18: Performance test for newly designed controller $C_{i+1}$

The new controller performs better than the old one for the Dual-Youla description, as the structured singular values of $M$ are lower than before. Also robust performance has been achieved, because $\mu_{\Delta}(M) \leq 1$. To achieve this performance level, the structured singular values were raised in the high frequent region so that they can be lowered for
low frequencies. In theory, the optimal structured singular value plot would be flat. The cause of this effect is related to the sensitivity integral and is also known as the 'waterbed effect'. This effect can also be seen in the additive structured singular values. The highest singular value for the additive uncertainty description, using the initial controller was at a low frequency, while for the Dual-Youla the bottleneck was near the bandwidth. The new controller was designed for the Dual-Youla case, so the low frequent part of the structured singular value has increased to decrease the part near the bandwidth. As a consequence the additive structured singular values increased even more which means the performance is worse for the newly designed controller.
6 Conclusion

6.1 Main conclusion

In this thesis a robust modeling approach and a performance robustness controller design for two slightly different tape drives is done to illustrate a way to deal with modeling errors in the system identification as well as a way to capture different systems within one set. In this way one can enhance performance on all of the systems within the set using validation tests of low complexity.

Closed loop system identification techniques were performed on the two LTO tape drives in order to get plant models. The measurement data was used to choose a nominal model and construct uncertainty models.

This thesis shows the benefits of a robust control design over a classic loop-shaping method when accounting for several systems at once. The robust control design is done with two different uncertainty descriptions. The first one is additive uncertainty, which uses only measured plant data and a nominal plant. The second one uses the Dual-Youla factorization in the uncertainty model to use the data of a known and stabilizing controller to construct a different set of models in a non-conservative way. Results show that due to the conservatism difference, one can guarantee a higher performance of a newly designed controller when using the Dual-Youla uncertainty description in comparison with the additive description. The choices during the design process, such as the nominal model choice and the choice of the weighting filters can significantly influence the final performance of the controller.

This shows that when using unstructured uncertainty modeling, controller data has to be used in order to prove robust stability and to achieve high performance for the constructed set of models. When one prefers to use additive uncertainty, a structural approach of the uncertainty modeling should be followed, in order to capture the dominant resonance peaks of the tape drives.

6.2 Recommendations

This thesis was done in a short timeframe and with only little available data. Therefore many aspects that could have contributed to higher performance of the feedback controller have not been considered in this thesis. The following remarks can be given.
• To construct a more accurate uncertainty model, the dynamics of more tape drives should be measured and taken into account. This way a better distinction can be made between measurement noise and model uncertainty, especially in the case of small resonance peaks around the bandwidth. Also more common structures could be identified and captured in the nominal model to reduce the size of the model set. This way the uncertainty set will be less conservative.

• The full use of co-prime factorizations was not used in this thesis. One could also consider choosing different factorizations to influence certain parts of the uncertainty description in a favorable way. Since this makes the performance optimization more complex this option was avoided in the thesis.

• Instead of using traditional loop shaping technique’s to design the new controller, one could use an H-infinity or DK-iteration approach, in which a cost function for the performance is minimized. The complexity of the controller might increase so model reduction techniques should be applied after finding the optimal controller. A lot of examples for the application of this can be found in the literature.

• Another way to characterize the uncertainty around a nominal model would be a structured uncertainty approach, which can also be used in combination with the unstructured method. One benefit of the use structured uncertainty is that one can specify exactly which models are captured in the uncertainty set. Furthermore, it results in a structured perturbation, which can reduce the structured singular value. This technique can also be favorable in combination with H-infinity synthesis or DK-iteration, because of the low order of model sets. The downside is that the control engineer needs to know a lot about the system to design a parametric model of the uncertainty. A structured uncertainty approach for magnetic recording devices is proposed in (de Callafon and Horowitz 2006).

• Also the newly high performance controller could be used to construct another set of models, in order to achieve even higher performance. The controller could also be used as a basis for adaptive control. This way the controller could adapt to the disturbances that are present in the system.

• Finally, the nonlinear behavior of the plant should be further investigated to confirm the earlier made assumption that the system will operate in the linear region.
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