Synergy between geo- and biomechanics

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Today, the focus of physical scientists is shifting to biology more than ever before. Because there is no biological tissue that is not a porous medium, the porous media community should be very alert to this shift of attention. The impact that porous media mechanics had on geosciences in the past centuries, may very well be reiterated in the field of biology - in an amplified form. The characteristic pore size in biological applications is close to the molecular level and hence below the Debye-Hueckel scale. Not only pressure gradients and concentration gradients, but electrical gradients as well are intimately linked to fluid flow, ion flow and deformation. To give a feeling for the subject, applications are covered in this presentation: blood perfusion, the mechanosensory mechanics in bone, the swelling and tearing of tissues.

1 INTRODUCTION
While the 20th century was the century of the physical sciences, the 21st century is expected to become the century of biology. The uncovering of the human genetic code is now followed by the uncovering of the mechanisms through which the genetic information is translated into the macroscopic human body. As all the functional constituents are present already at the level of microsized cell, many of the challenges lie at the nano- and microscale and require therefore advanced technical skills for both measuring and modelling. Even on the level of a single cell, the number of molecules involved are so high that if any modelling is going to be successful, it necessarily involves continuum mechanics. We will discuss one application on the microporescale, blood perfusion, and two applications on the nanoporescale, bone remodelling and the biomechanics of swelling.

2 BLOOD PERFUSION
Wilson, Aifantis and Barenblatt and many other have been successful in using methods of multiporosity porous media mechanics on fractured rocks (Wilson and Aifantis 1982). Multiporosity theory has applications even much closer to us. Coronary artery disease is believed to be number one cause of mortality in the world. The coronary vascular system is nothing but a pore structure inside a deforming solid which we call the heart muscle. The muscle is subject to large deformations. The pore structure is organised to deliver every heart cell from its waste materials and supply every cell of its nutrients. The way this is done is through a multiporosity structure (fig. 1). Since many centuries these porosities have names: arteries, arterioles, capillaries, venules and veins. They have their characteristic pore size, their characteristic blood velocity, their characteristic wall properties and their characteristic pathology. Every doctor in the world even knows a quantity proper to multiporosity flow: perfusion. It is the flow of fluid from one porosity to another, measured per unit volume tissue. Flow has a dimension $m^3s^{-1}$ and as it is defined per

![Figure 1: Corrosion cast of the coronary arterial tree of a canine heart (cast is courtesy of dr. P. Santens, University of Gent)](image-url)
Figure 2: Simulated blood pressure distribution across a section $\alpha - \alpha$ of the heart wall. During diastole most of the arteriovenous pressure drop is located in the arterioles. During systole the deeper endocardial layers of the heart wall are subjecting the coronary system to high pressure because of the strong muscle contraction.

unit volume, perfusion has a dimension of $1/s$. There is evidence that blood perfusion is affected by the deformation of the heart muscle (Spaan 1985). There is evidence that blood perfusion affects the deformation, both metabolically as mechanically (Olsen et al. 1981). These findings imply that the coronary system should be modelled using a multiporosity, finite deformation approach (fig. 2).

The multiporosity model was implemented in 2D, 3D and axisymmetric finite elements (Vankan et al. 1997). Animal experiments were undertaken to verify the concepts (van Donkelaar et al. 2001). Comparison of experiment and model clearly demonstrate the capabilities. The same equations were derived twice: using mixture theory (Vankan et al. 1996) and using averaging theorems (Huyghe and van Campen 1995a; Huyghe and van Campen 1995b). A micro-macro transformation was derived to compute hydraulic permeabilities from the microstructure (Huyghe et al. 1989; Huyghe et al. 1989; Vankan et al. 1997). While at the time of the research, the microstructure was hard to reconstruct, today micro-computed tomography are operational methods to reconstruct the 3D microgeometry post mortem.

Bone is known to adapt its pore structure to local mechanical load (fig. 3). Load-related alignment and mass of bone are the foundations of the Law of Bone Transformation (Wolff’s Law), formulated by Julius Wolff in 1892. Computer simulations confirmed his postulate, that bone adapts its form according to rules of mathematical design (Huiskes et al. 2000). However, the cellular mechanism that underlies the typical secondary bone structure is as yet unexplained. Bone remodelling involves groups of cells of 3 different types, which collaborate in so-called basic multi-cellular units: osteoblasts, osteoclasts and osteocytes Basic multicellular units proceed by tunnelling, during which osteoclasts excavate a canal that is partly refilled by osteoblasts, thus forming an osteon (fig. 4). The osteocytes reside in lacunae inside the bone matrix and thus have a good position for mechano-sensing. With their long slender protrusions that run through canaliculi they form a three-dimensional network that reaches to the bone surface, which allows them to signal (pre-) osteoclasts
Figure 4: Schematic view of an osteon. The vascular porosity contains osteoclasts (cells destroying bone mass) and osteoblasts (cells constructing bone mass). In the surrounding porous bone the osteocytes reside. They are interconnected by the lacuno-canalicular network (nano pore size).

and (pre-) osteoblasts. In order to respond to mechanical stimuli, bone necessarily needs to sense the mechanical load it adapts to. The most accepted hypothesis on how bone senses its own stress field, involves fluid flow induced by mechanical load (Cowin et al. 1991). In other words it involves a consolidation process, similar to the consolidation of geomaterials. Burger and Klein Nulend (Burger and Nulend 1999) have demonstrated through several experimental studies in vitro that osteocytes react to fluid flow. As an early response they release nitric oxide (NO) and prostaglandins (PG) E2 and I2, followed by expression of the enzyme cyclo-oxygenase 2 (COX-2), which allows for continued release of prostaglandins. NO, PGE2 and COX-2 play a crucial role in the induction of bone formation (Chambers et al. 1999), while NO and PGE2 also inhibit osteoclasts. Large tissue strains thus conceivably lead to osteoblast recruitment. Conversely, decreased stimulation of osteocytes may activate osteoclasts, possibly by the mechanism of cell death. As osteocytes are likely stimulated by strain-induced fluid flow, my colleagues of the Free University of Amsterdam and myself aimed to determine the local pattern of fluid flow at a remodelling site (Smit et al. 2002).

Cortical bone has two systems of interconnected channels. The largest of these is the vascular porosity consisting of Haversian and Volkmann’s canals, with a diameter of some 50 $\mu$m, which contains a.o. blood vessels and nerves. The smaller system consists of the canaliculi and lacunae. The canaliculi are at the submicron level and house the protrusions of the osteocytes. When bone is loaded, fluids within the solid matrix sustain a pressure gradient that drives a flow. It is generally assumed that the flow of extra-cellular fluid around osteocytes plays an important role not only in the nutrition of these cells, but also in the bone’s mechanosensory system. The interaction between the deformation of the bone matrix and the flow

Figure 5: Local fluid content change in an osteon during walking as computed by Biot’s theory. In front of the cutting cone appears an area of volumetric expansion that effectively inhibits the exchange of fluid between the bone matrix and the vascular porosity (Smit et al. 2002).

Figure 6: Measured versus simulated streaming potential across the shaft of an osteon. The consolidation time is in the order of magnitude of one walking cycle.
of fluid is modeled using Biot’s theory of poroelasticity. However, because of the inhomogeneity of the bone matrix and the scale of the porosities, it is not possible to experimentally determine all the parameters that are needed for numerical implementation. Smit et al. (Smit et al. 2002) derive these parameters using composite modelling and experimental data from literature. A full set of constants is estimated for a linear isotropic description of cortical bone as a two-level porous medium. An axisymmetric finite element model of the minerised bone surrounding a haversian canal is constructed. The lacuno-canalicular porosity is modelled as a pore space saturated with compressible fluid. The axial load depends on time according to the typical loading pattern of a femur during walking. Along the cylindrical part of the tunnel (the closing cone), fluid is pressed out of the bone matrix. At the tip (the cutting cone), however, fluid flows into the bone matrix, because of a local, superficial area of volumetric expansion. The amplitude of this fluid flow at the cutting cone is about six times lower than the flow at the closing cone. At unloading of the bone tissue, the fluid flow pattern is reversed, resulting in a fluid outflow at the cutting cone, and an inflow along the rest of the osteonic tunnel wall. Inside the bone matrix, the outflow pattern along the closing cone damps out at a depth of some 100 µm. This 100 µm is just about the distance from the deepest osteocytes to the bone surface both in human compact and trabecular bone, which confirms a role for the transport of nutrients. At the cutting cone, however, a different phenomenon is observed: here the fluid flow direction is reversed at a depth of some 10 µm, because the volumetric expansion is only a superficial phenomenon and the deeper layers experience volumetric compression (Smit et al. 2002). The area of volumetric expansion in front of the cutting cone not only reduces the flow amplitude around the osteocytes located there, but inhibits the exchange of fluid between the bone matrix and the osteonic lumen as well. The bone tissue layer immediately in front of the cutting cone thus appears as an area of local disuse with lack of fluid transport. This is precisely the bone layer that osteoclasts erode. By contrast, behind the cutting cone the tissue deformation and the canalicular fluid flow are increased; there the osteocytes are inhibited from further excavation, while osteoblasts are recruited in order to refill the tunnel. So by excavating a tunnel, osteoclasts create a local area of increased canalicular fluid flow that leads to osteoblast recruitment, and at the same time create an area of disuse in the loading direction that guides their continued resorption activity. This explains the progression of osteon formation along the principle loading direction (Smit et al. 2002). From the simulated flow pattern, the time course of the streaming potential across the shaft of the osteon is computed (fig. 6) assuming a vanishing streaming current. The hydraulic permeability is chosen so as to reproduce experimentally measured data from the literature (Otter et al. 1992). The above mechanism explains the orientation of the bone structure along the principle stress direction and the tuning of bone mass according to the magnitude of the stress and hence provides a cellular basis for Wolff’s law (Smit et al. 2002). In this mechanism, Biot’s theory is a key element, indicating the strong need for communication between the porous media community and biologists. Including streaming potentials, one finds a gradient in electrical potential within the haversian canal, which may play a role in the guiding of osteoclasts and osteoblasts.

4 SWELLING TISSUES AND CELLS

Since antiquity, the phenomenon of swelling of tissues has been closely related to health and disease. Biological, synthetic and mineral porous media often exhibit swelling or shrinking when in contact with changing salt concentrations. This phenomenon, observed in clays, shales, cartilage and gels, is caused by a combination of electrostatic forces and hydrostatic forces (Lai, Hou, and Mow 1991). In case of biological tissue, electrostatic forces are often dominant. Classical concepts, such as the transmembrane potential of cells are directly associated with these electrostatic forces. Already years ago, Biot understood that his theories were closely associated with transmembrane phenomena in living cells (). At least four components are involved in the swelling mechanics: a solid, a fluid, anions and cations. Lai et al. (Lai, Hou, and Mow 1991) developed a triphasic theory for soft hydrated tissue and applied the theory to cartilage while neglecting geometric non-linearities. They verified the theory for one-dimensional equilibrium results. As soft tissues and cells are commonly subject to large deformations, our group developed a finite deformation theory of ionised media (Huyghe and Janssen 1997). In order to simplify the mathematics as much as possible a Lagrangian form of the entropy inequality has been derived which leads to equations consistent with Biot’s porous media theories in a more straightforward way than the more familiar Eulerian approach of Bowen (Bowen 1980). The incompressibility and electroneutrality conditions are introduced by means of two Lagrange multipliers; the latter is physically interpreted as an electrical potential, the former as a pressure. The intervertebral disc is a classical example of a swelling tissue. Excised from the spine, the intervertebral disc swells to three times its volume in physiological salt solution. The disc has therefore a strong pre-stressing within the spine, as the swelling propensity is contained. The degeneration of the disc in the mid-
dle aged adult, results in the development of cracks in the tissue, that have very poor correlation to me-
chanoical load. The tearing of the disc and the coupled degeneration seems to be orchestrated as a intrinsic aging process associated with specific genes.

5 Donnan Osmosis

When an ionised medium is in contact with a mono-
valent salt solution, diffusion of salt ions and flow of fluid take place between the medium and the salt so-
lution until equilibrium is reached:

\[ \mu^+ = \bar{\mu}^+ \]  
(1) 

\[ \mu^- = \bar{\mu}^- \]  
(2) 

\[ \mu^f = \bar{\mu}^f \]  
(3)

\( \mu^+ \) is the electrochemical potential of the cations, \( \mu^- \) is the electrochemical potential of the anions and \( \mu^f \) the chemical potential of the fluid in the medium. The corresponding overlined symbols refer to chemical potentials in the outer solution. Standard expressions for (electro)chemical potentials are found in the literature (Richards 1980). If we assume incompressibility for each constituent, i.e. same partial molar volumes in either solution, we find:

\[ \mu^+ = \mu_0^+ + \frac{1}{V^+}(RT\ln a^+ + F\xi) \]  
(4) 

\[ \mu^- = \mu_0^- + \frac{1}{V^-}(RT\ln a^- - F\xi) \]  
(5) 

\[ \mu^f = \mu_0^f + p + \frac{RT}{V^f}\ln a^f \]  
(6)

in which \( \mu_0^\beta \) are reference values, \( V^\beta \) partial molar volumes, \( a_0^\beta \) activities, \( p \) the fluid pressure, \( T \) abso-
lute temperature, \( R \) universal gas constant, \( F \) Faraday’s constant and \( \xi \) the electrical potential. All of these (electro)chemical potentials are measured here per unit volume constituent. Combination of equation (1) and (2) leads to:

\[ a^- a^+ = a^+ a^- \]  
(7) 

\[ \xi - \bar{\xi} = \frac{RT}{2F\ln a^- a^+} \]  
(8)

where \( \xi - \bar{\xi} \) is the Donnan potential between the inner and outer solution. If we define \( \phi^c \) as the fixed charge density per unit fluid volume of the inner solution, taken positive for positive charges and negative for negative charges, we can write the electroneutrality conditions as:

\[ c^- = c^+ + \phi^c \]  
(9)

c+ and c are the cationic and anionic concentrations per unit fluid volume in the inner solution, while the corresponding overlined symbols pertain to the outer solution. From the previous equations we derive the Donnan equilibrium concentration of the ions:

\[ 2c^+ = -c^f + \sqrt{(c^f)^2 + 4f^2\bar{\mu}^2} \]  
(11)

\[ 2c^- = c^f + \sqrt{(c^f)^2 + 4f^2\bar{\mu}^2} \]  
(12)

with

\[ f^2 = \frac{\bar{f}^+ f^-}{\bar{f}^- f^+} \]  
(13)

and \( f^\beta = \frac{a^\beta}{a^\beta_0} \), \( \beta = +, - \) the activity coefficient of component \( \beta \). Eqs. (11-12) show that the cationic concentration jumps to a higher and the anionic concentration to a lower value when entering the porous medium. These concentration jumps are responsible for the attraction of water into the porous medium during swelling and for the associated osmotic pressure \( \pi \). Using eq. (6) one can derive Van 't Hoff relation from (3):

\[ \pi = p - \bar{p} = RT[\phi^f (c^+ + c^-) - 2\phi^f \bar{\mu}] \]  
(14)

provided that the molar fractions of the ions are small compared to the molar fraction of the fluid. \( \phi^f \) and \( \bar{\phi}^f \) are the osmotic coefficients:

\[ \phi^f = \frac{\partial\ln a^f}{\partial\ln x^f} \]  
(15)

\[ \bar{\phi}^f = \frac{\partial\ln a^f}{\partial\ln \bar{x}^f} \]  
(16)

6 Crack propagation in Donnan osmotic medium

Figure 7: Partition of Unity simulation of a crack propagation in a swelling medium. From left to right: no prestressing, prestressing in vertical direction, prestressing in horizontal direction and prestressing in both direction.

Our present efforts of research concentrate on the modelling of the heterogeneous prestressing of the
disc during degeneration, given a fixed charge density that decreases with time. A meshfree Partition of Unity description of the crack propagation is chosen in a poroelastic medium in which Donnan osmotic swelling is incorporated. As an example crack propagation is shown in Fig. 7.

7 CONCLUSIONS
The application of porous media mechanics in biology is growing field of interest, that presents challenges which in many aspects are similar to those encountered in geomechanics. At small pore scales, electrical effects become very significant. However, the biological materials have a structure that is far more sophisticated. Hence, cooperation with many more disciplines is mandatory in order to cover the whole range expertise needed to address all issues. It is vital that the community of porous media experts contribute their share to understanding of our own bodies and those of other species.

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