Robust stability assessment in High-speed milling

Niels van Dijk*, Nathan van de Wouw and Henk Nijmeijer
Department of Mechanical Engineering, Eindhoven University of Technology
P.O. Box 513, 5600 MB Eindhoven, The Netherlands
Email: *N.J.M.v.Dijk@tue.nl

1 Introduction

In high-speed milling (HSM), the occurrence of chatter vibrations limits the performance of the milling process. The occurrence of chatter results in heavy vibrations of the cutter, increased wear of the cutter, noise and an inferior work-piece quality, see Figure 1. Stability of the milling process is characterised in so-called Stability Lobe Diagrams (SLD) in which the stability boundary is given in terms of two process parameters: the spindle-speed $n \propto 1/\tau$, with delay $\tau$, and depth of cut $a_p$.

Figure 1: Workpiece without (left) and with (right) chatter marks.

2 Approach

Here, a method is presented to determine robust stability for a certain range in spindle speeds. The uncertain system is represented as a feedback interconnection between the nominal milling system, using a linear cutting force model, $F = a_p H(v(t) - v(t - \tau))$, and machine dynamics model, $\dot{x} = Ax(t) + BF(t)$, $v(t) = Cx(t)$, see [1], yielding

$$\dot{x}(t) = A_0 x(t) + A_1 x(t - \tau),$$

$$v(t) = C x(t),$$

where $A_0 = A + a_p BH$, $A_1 = -a_p BH$, and uncertainties $\delta a_p$ and $\delta \tau$, in the depth-of-cut and delay, respectively. Using stability characterisations as in [2], it can be concluded that the system is robustly stable if

$$\|\Delta\|_{\infty} \leq (\sup_{\omega>0} \mu_{\Delta} P(j\omega))^{-1},$$

where $\mu_{\Delta}$ is the structured singular value of

$$P(s) = \begin{bmatrix} C s & C s (1 - e^{-s\tau}) \end{bmatrix} \left[ s I - A_0 - A_1 e^{-s\tau} \right]^{-1} \begin{bmatrix} a_p s H & \frac{1}{w_2} p H \end{bmatrix} + \begin{bmatrix} 0 & 0 \\
0 & 1 \\ -1 \\
0 \end{bmatrix} \begin{bmatrix} 1 \\
\tau \\
\delta \tau \\
\delta a_p \end{bmatrix},$$

with respect to uncertainty

$$\Delta(s) = \begin{bmatrix} w_1 e^{-s(1 - e^{-s\delta\tau})} I & 0 \\
0 & w_2 \delta a_p I \end{bmatrix}.$$ 

Weighting factors are chosen as $w_1 = 1/\delta \tau_{\max}$ and $w_2 = 1/\delta a_{p,\max}$ with $\delta \tau_{\max}$ and $\delta a_{p,\max}$ indicating the area in the SLD for which stability should be assessed.

3 Results

Norm-bounded stability criteria, as used above, introduce conservativeness. Therefore, the approach is compared to a SLD determined via the semi-discretisation method, see [1]. Results are depicted in Figure 2 for $w_1 = 4.0945 \cdot 10^4$, $w_2 = 1.040$. It can be seen that the proposed strategy very well predicts a stability region, i.e. the rectangle (almost) touches the SLD at the corner points. Moreover, since there is no need to calculate the entire SLD this strategy for robust stability analysis is computationally efficient.

Figure 2: SLD and calculated norm-based stability region.

4 Conclusions

An approach for robust stability assessment for the high-speed milling process is presented. Future work includes the development of controllers that stabilise a predefined region in the stability lobes diagram.

References
